



<b>EXPANDING BASED ON BINOMIAL THEOREM</b>	
Q.9)	Expand using binomial theorem expansion $\left(\frac{2}{x} - \frac{x}{2}\right)^5$ ?
Sol.9)	<p>Given expansion: <math>\left(\frac{2}{x} - \frac{x}{2}\right)^5</math></p> $= 5C_0 \left(\frac{2}{x}\right)^5 \left(\frac{x}{2}\right)^0 - 5C_1 \left(\frac{2}{x}\right)^4 \left(\frac{x}{2}\right)^1 + 5C_2 \left(\frac{2}{x}\right)^3 \left(\frac{x}{2}\right)^2 - 5C_3 \left(\frac{2}{x}\right)^2 \left(\frac{x}{2}\right)^3 + 5C_4 \left(\frac{2}{x}\right)^1 \left(\frac{x}{2}\right)^4 - 5C_5 \left(\frac{2}{x}\right)^0 \left(\frac{x}{2}\right)^5$ $= \left(\frac{32}{x^5}\right)^1 (1) - 5 \left(\frac{16}{x^4}\right) \left(\frac{x}{2}\right) + 10 \left(\frac{8}{x^3}\right) \left(\frac{x^2}{4}\right) - 10 \left(\frac{4}{x^2}\right) \left(\frac{x^3}{8}\right) + 5 \left(\frac{2}{x}\right) \left(\frac{x^4}{16}\right) + (1)(1) \left(\frac{x^5}{32}\right)$ $= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^4}{8} - \frac{x^3}{32} \text{ ans.}$
Q.10)	Evaluate $(99)^5$ using binomial theorem ?
Sol.10)	<p>We have, <math>(99)^5 = (100 - 1)^5</math></p> $= 5C_0 (100)^5 (1)^0 - 5C_1 (100)^4 (1)^1 + 5C_2 (100)^3 (1)^2 - 5C_3 (100)^2 (1)^3 + 5C_4 (100)^1 (1)^4 - 5C_5 (100)^0 (1)^5$ $= 100000000000 - 5(100000000) + 10(1000000) - 10(10000) + 5(100) - 10$ $= (100000000000 + 100000000 + 500) - (500000000 + 100000 + 1)$ $= 10010000500 - 500100001$ $= 9509900499 \text{ ans.}$
Q.11)	Find $(a + b)^4 - (a - b)^4$ , hence evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$ ?
Sol.11)	$(a + b)^4 - (a - b)^4$ $= [4C_0 a^4 b^0 + 4C_1 a^3 b^1 + 4C_2 a^2 b^2 + 4C_3 a^1 b^3 + 4C_4 a^0 b^4] - [4C_0 a^4 b^0 - 4C_1 a^3 b^1 + 4C_2 a^2 b^2 - 4C_3 a^1 b^3 + 4C_4 a^0 b^4]$ $= 4C_1 a^3 b^1 + 4C_3 a^1 b^3 + 4C_1 a^3 b^1 + 4C_3 a^1 b^3$ $= 2(4C_3 a^1 b^3) + 2(4C_1 a^3 b^1)$ $= 8a^3 b^1 + 8a^1 b^3 \quad \dots \{4C_3 = 4C_1 = 4\}$ $= 8ab(a^2 + b^2)$ $\therefore (a + b)^4 - (a - b)^4 = 8ab(a^2 + b^2)$ <p>Now, for <math>(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4</math>, put <math>a = \sqrt{3}</math> &amp; <math>b = \sqrt{2}</math></p> $= (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8\sqrt{3}\sqrt{2}(3 + 2)$ $= 8\sqrt{6}(5) = 40\sqrt{6} \text{ ans.}$
Q.12)	Which is larger $(1.1)^{10000}$ or 1000?
Sol.12)	$(1.1)^{10000} = (1 + 0.1)^{10000}$ $= {}^{10000}C_0 (0.1)^0 10000 C_1 (0.1)^1 + \dots$ $= (1)(1) + 10000(0.1) + \dots$ $= 1 + 1000 + \dots \text{ (+) positive term}$ $= 10001 + \dots > 1000$ $\therefore (1.1)^{10000} \text{ is larger than } 1000 \text{ ans.}$
Q.13)	Prove that $\sum_{r=0}^n 3^r \cdot nC_r = 4^n$ ?
Sol.13)	<p>L.H.S. <math>\sum_{r=0}^n 3^r \cdot nC_r</math></p> $= 3^0 nC_0 + 3^1 nC_1 + 3^2 nC_2 + \dots 3^n nC_n \dots \quad (1)$ <p>R.H.S. <math>4^n = (1 + 3)^n</math></p> $= nC_0 3^0 + nC_1 3^1 + nC_2 3^2 + \dots 3^n nC_n \dots \quad (2)$ <p>From (1) and (2), L.H.S = R.H.S. (proved)</p>
Q.14)	Prove that $6^n - 5n$ always leaves the remainder 1, when divided by 25?

Sol.14)	$  \begin{aligned}  6^n - 5n &= (1+5)^n - 5n \\  &= (n_{C_0}5^0 + n_{C_1}5^1 + n_{C_2}5^2 + \dots + n_{C_n}5^n) - 5n \\  &= (1 + 5n + n_{C_2}5^2 + n_{C_3}5^3 + \dots + 5^n) - 5n \\  &= 1 + n_{C_2}5^2 + n_{C_3}5^3 \dots 5^n \\  &= 6^n - 5n = 1+25(n_{C_2} + 5.n_{C_2} + \dots + 5^{n-2})  \end{aligned}  $ <p>Clearly it leaves the remainder 1 when divided by 25</p>
Q.15)	Show that $9^{n+1} - 8n - 9$ is divisible by 64?
Sol.15)	$  \begin{aligned}  9^{n+1} - 8n - 9 &= (1+8)^{n+1} - 8n - 9 \\  &= {}^{n+1}C_08^0 + {}^{n+1}C_18^1 + {}^{n+1}C_28^2 + {}^{n+1}C_38^3 + \dots 8^{n+1} - 8n - 9 \\  &= (8n + 9 + n + 1{}^{n+1}C_28^2 + n + 1{}^{n+1}C_38^3 + \dots 8^{n+1}) - 8n - 9 \\  &= {}^{n+1}C_28^2 + {}^{n+1}C_38^3 + \dots 8^{n+1} \\  &= 8^2(n + 1{}^{n+1}C_28^2 + n + 1{}^{n+1}C_38^3 + \dots 8^{n+1})  \end{aligned}  $ <p>Clearly it is divisible by 64 ans.</p>
Q.16)	Prove that $(a-b)$ is a factor of $a^n - b^n$ ?
Sol.16)	$  \begin{aligned}  a^n - b^n &= [(b + (a-b))]^n - b^n \\  &= n_{C_0}b^n(a-b)^0 + {}^nC_1b^{n-1}(a-b)^1 + {}^nC_2b^{n-2}(a-b)^2 + \dots + {}^nC_nb^0(a-b)^n - b^n \\  &= b^n + n.b^{n-1}(a-b) + {}^nC_2b^{n-2}(a-b)^2 + \dots + (a-b)^n - b^n \\  &= nb^{n-1}(a-b) + {}^nC_2b^{n-2}(a-b)^2 + \dots + (a-b)^n \\  &= (a-b)[nb^{n-1} + {}^nC_2b^{n-2}(a-b)^2 + \dots + (a-b)^{n-1}]  \end{aligned}  $ <p>Clearly it is divisible by <math>(a-b)</math> ans.</p>
Q.17)	If 'O' be the sum of odd terms & 'E' be the sum of even terms in the expansion of $(x+a)^n$ prove that, $O^2 - E^2 = (x^2 - a^2)^n$
Sol.17)	$  \begin{aligned}  (x+a)^n &= n_{C_0}x^n a^0 + n_{C_1}x^{n-1}a^1 + n_{C_2}x^{n-2}a^2 + \dots + n_{C_n}x^0 a^n \\  \Rightarrow (x+a)^n &= (n_{C_0}x^n a^0 + n_{C_2}x^{n-2} + n_{C_4}x^{n-4}a^4 + \dots) + (n_{C_1}x^{n-1}a^1 + \\  &\quad n_{C_3}x^{n-3}a^3 + n_{C_5}x^{n-5}a^5 + \dots) \\  \Rightarrow (x+a)^n &= O + E \dots (1)  \end{aligned}  $ <p>Now, <math>(x-a)^n = n_{C_0}x^n a^0 - n_{C_1}x^{n-1}a^1 + n_{C_2}x^{n-2}a^2 - \dots + n_{C_n}x^0 a^n</math></p> $  \begin{aligned}  &= (n_{C_0}x^n a^0 + n_{C_2}x^{n-2}a^2 + n_{C_4}x^{n-4}a^4 + \dots) - (n_{C_1}x^{n-1}a^1 + \\  &\quad n_{C_3}x^{n-3}a^3 + n_{C_5}x^{n-5}a^5 + \dots) \\  \Rightarrow (x-a)^n &= O - E \dots (2)  \end{aligned}  $ <p>Multiply eq. (1) &amp; eq. (2)</p> $  \begin{aligned}  (x+a)^n(x-a)^n &= (O+E)(O-E) \\  \Rightarrow ((x+a)(x-a))^n &= O^2 - E^2 \\  \Rightarrow (x^2 - a^2)^n &= O^2 - E^2 \text{ (proved)}  \end{aligned}  $
Q.18)	Find the coefficient of $a^4$ in the product $(1+2a)^4(2-a)^5$ using binomial theorem?
Sol.18)	<p>Since, expansion are in product  <math>\therefore</math> general term cannot be formed</p> $  \begin{aligned}  (1+2a)^4(2-a)^5 &= [4_{C_0}(2a)^0 + 4_{C_1}(2a)^1 + 4_{C_2}(2a)^2 + 4_{C_3}(2a)^3 + 4_{C_4}(2a)^4] \\  &\quad \times [5_{C_0}(2)^5(a)^0 - 5_{C_1}(2)^4(a)^1 + 5_{C_2}(2)^3(a)^2 - 5_{C_3}(2)^2(a)^3 \\  &\quad + 5_{C_4}(2)^1(a)^4 - 5_{C_5}(2)^0(a)^5] \\  &= (1+8a+24a^2+32a^3+160a^4) \times (32-80a+80a^2-40a^3+10a^4-a^5)  \end{aligned}  $ <p>The terms containing <math>a^4</math> are</p> $  \begin{aligned}  &= 1(10a^4) + (8a)(-40a^3) + (24a^4)(80a^2) + (32a^3)(-80a) + (16a^4)(32) \\  &= 10a^4 - 320a^4 + 1920a^4 - 2560a^4 + 51a^4 = -438a^4 \\  \therefore \text{coefficient of } a^4 &= -438 \text{ ans.}  \end{aligned}  $
Q.19)	Expand using binomial theorem $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4$ ?

Sol.19)	$  \begin{aligned}  (1 + \frac{x}{2} - \frac{2}{x})^4 &= \left[1 + \left(\frac{x}{2} - \frac{2}{x}\right)\right]^4 \\  &= 4C_0 \left(\frac{x}{2} - \frac{2}{x}\right)^0 + 4C_1 \left(\frac{x}{2} - \frac{2}{x}\right)^1 + 4C_2 \left(\frac{x}{2} - \frac{2}{x}\right)^2 + 4C_3 \left(\frac{x}{2} - \frac{2}{x}\right)^3 + 4C_4 \left(\frac{x}{2} - \frac{2}{x}\right)^4 \\  &= (1)(1) + 4\left(\frac{x}{2} - \frac{2}{x}\right) + 6\left(\frac{x^2}{4} - \frac{4}{x^2} - 2\right) + 4\left[\frac{x^3}{8} - \frac{8}{x^3} - 3\left(\frac{x^2}{4}\right)\left(\frac{2}{x}\right)\right] + \\  &\quad 3\left(\frac{x}{2}\right)\left(\frac{4}{x^2}\right)\left[4C_0 \left(\frac{x}{2}\right)^0 \left(\frac{2}{x}\right)^0 - 4C_1 \left(\frac{x}{2}\right)^1 \left(\frac{2}{x}\right)^1 + 4C_2 \left(\frac{x}{2}\right)^2 \left(\frac{2}{x}\right)^2 - 4C_3 \left(\frac{x}{2}\right)^3 \left(\frac{2}{x}\right)^3 + \right. \\  &\quad \left. 4C_4 \left(\frac{x}{2}\right)^6 \left(\frac{2}{x}\right)^4\right] \dots \dots \dots \text{in } \left(\frac{x}{2} - \frac{2}{x}\right)^4 \text{ we use again binomial theorem} \\  &= 1 + 2x - \frac{8}{x} + \frac{3x^2}{2} + \frac{24}{x^2} - 12 + \frac{x^3}{2} - \frac{32}{x^2} - 6x + \frac{24}{x} + \frac{x^4}{16} - x^2 + 6 - \frac{16}{x^2} + \frac{16}{x^4} \\  &= \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4} - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - 5 \text{ ans.}  \end{aligned}  $
Q.20)	If 'O' be the sum of odd terms & 'E' be the sum of even terms in the expansion of $(x + a)^n$ prove that, 1. $4OE = (x + a)^{2n} - (x - a)^{2n}$ 2. $2(O^2 - E^2) = 1(x + a)^{2n} - (x - a)^{2n}$
Sol.20)	<ol style="list-style-type: none"> <li>We know that, <math>4OE = (O + E)^2 - (O - E)^2</math>  <math>\Rightarrow 4OE = [(x + a)^n]^2 - [(x - a)^n]^2 \dots \dots \text{using eq. (1) and eq. (2)}</math>  <math>\Rightarrow 4OE = (x + a)^{2n} - (x - a)^{2n} \text{ (proved)}</math>            Squaring eq. (1) &amp; eq. (2)         </li> <li>We get, <math>(x + a)^{2n} = (O + E)^2 \dots \dots \text{(3)}</math>            And <math>(x - a)^{2n} = (O - E)^2 \dots \dots \text{(4)}</math>            Now, eq. (3) + (4)  <math>\Rightarrow (x + a)^{2n} - (x - a)^{2n} = 2(O^2 - E^2) \text{ (proved)}</math> </li> </ol>