	BINOMIAL THEOREM
Q.1)	The 2nd, 3rd & 4th terms in the expansion of $(x+a)^n$ are 240, 720 & 1080. Find x, a & n?
Sol.1)	Given expansion: $(x+a)^n$ $T_2 = 240, T_3 = 720 \;\&\; T_4 = 1080$ General term: $T_{r+1} = {}^n c_r x^{n-r} a^r$ Now, $T_2 = {}^n c_1 x^{n-1} a^1 = 240$ .... (1) = $T_3 = {}^n c_2 x^{n-2} a^2 = 720$ .... (2) = $T_4 = {}^n c_3 x^{n-3} a^3 = 1080$ .... (3) Equation (2) ÷ equation (1) $\Rightarrow \frac{{}^n c_2 x^{n-2} a^2}{{}^n c_1 x^{n-1} a^1} = \frac{720}{240} = 3$ $\Rightarrow \frac{\frac{n(n-1)}{2}.x^{n-2+n+1}.a}{n} = 3$ $\Rightarrow \frac{(n-1)x^{-1}a}{2} = 3$ $\Rightarrow \frac{(n-1)a}{2x} = 3$ $\Rightarrow (n-1)a = 6x$ .... (4) Now, equation (3) ÷ equation (2) $\Rightarrow \frac{{}^n c_3 x^{n-3} a^3}{{}^n c_2 x^{n-2} a^2} = \frac{1080}{720}$ $\Rightarrow \frac{\frac{n(n-1)(n-2}{6}.x^{n-3-n+2}.a}{\frac{(n-1)}{2}} = \frac{3}{2}$ $\Rightarrow \frac{(n-2).x^{-1}a}{3} = \frac{3}{2}$ $\Rightarrow \frac{(n-2)a}{3x} = \frac{3}{2}$ $\Rightarrow 2(n-2)a = 9x$ .... (5) Now, equation (5) ÷ equation (4) $\Rightarrow \frac{2(n-1)a}{(n-1)a} = \frac{9x}{6x}$ $\Rightarrow \frac{2n-4}{n-1} = \frac{3}{2}$ ⇒ 4n-8 = 3n-3 ⇒ n=5, put this value in equation (4) We get, $4a=6x$ $\Rightarrow a = \frac{3x}{2}$, put value of n & a in equation (1) We have, $5c_1(x)^4.\left(\frac{3x}{2}\right) = 240$ $\Rightarrow 5x^5\frac{3}{2} = 240$ $\Rightarrow x^5 = \frac{240\times2}{15}$ $\Rightarrow x^5 = 32 = 2^5$ $\Rightarrow x = 2, a = 3$ .... $\left\{since\; a = \frac{3x}{2}\right\}$ $\therefore n = 5, x = 2\; and\; a = 3$ ans.
Q.2)	Find a, b & n in expansion of $(a+b)^n$, if the first three terms in the expansion are 729, 7290 & 30375?
Sol.2)	Given expansion: $(a+b)^n$ $T_1 = 729, T_2 = 7290 \;\&\; T_3 = 3037$ General terms : $T_{r+1} = {}^n c_r a^{n-r} b^r$ Now, $T_1 = {}^n c_0 a^n b^0 = 729$ $\Rightarrow T_1 = a^n = 729$ .... (1) $\Rightarrow T_2 = {}^n c_1 a^{n-1} b^1 = 7290$ .... (2)

$\Rightarrow T_3 = {}^nc_2 a^{n-2} b^2 = 30375$ .... (3)

Now, equation (2) ÷ equation (1)

$\Rightarrow \frac{{}^nc_1 a^{n-1} b^1}{a^n} = \frac{7290}{729}$

$\Rightarrow n.a^{n-1-n}.b = 10$

$\Rightarrow \frac{nb}{a} = 10$

$\Rightarrow nb = 10a$ .... (4)

Now, equation (3) ÷ equation (2)

$\Rightarrow \frac{{}^nc_2 a^{n-2} b^2}{{}^nc_1 a^{n-1} b^1} = \frac{30375}{7290}$

$\Rightarrow \frac{\frac{n(n-1)}{2}.a^{n-2-n+1}.b}{n} = \frac{25}{6}$

$\Rightarrow \frac{\frac{(n-1)}{2}.a^1.b}{n} = \frac{25}{6}$

$\Rightarrow \frac{(n-1)b}{2a} = \frac{25}{6}$

$\Rightarrow 6(n-1)b = 50a$ .... (5)

Now, equation (5) ÷ (4)

$\Rightarrow \frac{6(n-1)b}{nb} = \frac{50a}{10a}$

$\Rightarrow 6n - 6 = 5n$

$\Rightarrow n = 6$, put the value of n in equation (4)

$\Rightarrow 6b = 10a$

$\Rightarrow b = \frac{5a}{3}$

Now, from equation (1) put n=3

$\Rightarrow a^6 = 729$

$\Rightarrow a^6 = 3^6$

$\Rightarrow a = 3$

$\therefore n = 6, a = 3 \ \& \ b = 5$ ans.

Q.3) If the coefficient of a^{r-1}, a^r *and* a^{r+1} in the expansion of $(1+a)^n$ are in A.P, show that $n^2 - n(4r+1) + 4r^2 - 2 = 0$?

Sol.3) Given expansion: $(1+a)^n$

General term: $T_{r+1} = {}^nc_r (1)^{n-r} a^n$

$T_{r+1} = {}^nc_r a^r$

Clearly, coefficient of $a^r = {}^nc_r$

$\therefore$ coefficient of $a^{r-1} = {}^nc_{r-1}$

And coefficient of $a^{r+1} = {}^nc_{r+1}$

We are given that, ${}^nc_{r-1}$, ${}^nc_r$ and ${}^nc_{r+1}$ are in A.P

$\Rightarrow 2.{}^nc_r = {}^nc_{r-1} + {}^nc_{r+1}$

$\Rightarrow 2.\frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!} + \frac{n!}{(r+1)!(n-r-1)!}$

$\Rightarrow \frac{2}{r!(n-r)!} = \frac{1}{(r-1)!(n-r+1)!} + \frac{1}{(r+1)!(n-r-1)!}$

$\Rightarrow \frac{2}{r(r-1)!(n-r-1)!} = \frac{1}{(r-1)!(n-r+1)(n-r)(n-r-1)!} + \frac{1}{(r+1)r!(r-1)(n-r+1)!}$

$\Rightarrow \frac{2}{r(n-r)} - \frac{1}{(n-r+1)(n-r)} = \frac{1}{(r+1)r}$

$\Rightarrow \frac{2}{r(n-r)} - \frac{1}{(n-r+1)(n-r)} = \frac{1}{(r+1)}$

$\Rightarrow \frac{2(n-r+1)-r}{r(n-r)(n-r+1)} = \frac{1}{r+1}$

$\Rightarrow 2nr + 2n - 3r^2 - 3r + 2r + 2 = n^2 - nr + n - nr + r^2 - r$

$\Rightarrow n^2 - 4nr - n + 4r^2 - 2 = 0$

$\Rightarrow n^2 - n(4r+1) + 4r^2 - 2 = 0$ (proved)

Q.4) Find the 4[th] term from the end in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{11}$.

Sol.4)	Given expansion : $\left(x^4 - \frac{1}{x^3}\right)^{11}$ General terms: T_{r+1} = $(-1)^r 11_{C_r}(x^4)^{11-r}\left(\frac{1}{x^3}\right)^r$ $\Rightarrow T_{r+1}$ = $(-1)^r 11_{C_r}(x)^{44-4r}.\frac{1}{x^{3r}}$ $\Rightarrow T_{r+1}$ = $(-1)^r 11_{C_r}(x)^{44-7r}.\frac{1}{x^{3r}}$ Formula, r^{th} term from the end $= (n-1+2)^{th}$ term from beginning and 4^{th} term from the end $= (11-4+2)^{th}$ term from beginning = $9^{th} term.$ For T_9, put r = 8 $\Rightarrow T_9$ = $(-1)^8 11_{C_8}(x)^{44-56}$ $\Rightarrow T_9$ = $11_{C_3}(x)^{-12}.\frac{1}{x^{3r}}$ .... $\{n_{C_r} = n_{C_{n-r}}\}$ $\Rightarrow T_9$ = $\frac{11\times10\times9}{6}.\frac{1}{x^{12}}$ $\Rightarrow T_9$ = $\frac{165}{x^{12}}$ $\therefore 4^{th}$ term from the end $= \frac{165}{x^{12}}$ ans.	
Q.5)	Find the value of n , if the ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}: 1$.	
Sol.5)	Expansion: $\left(2^{1/4} + \frac{1}{3^{1/4}}\right)$ General term: T_{r+1} = $^nc_r\left(2^{1/4}\right)^{n-r}\left(\frac{1}{3^{1/4}}\right)^r$ $\Rightarrow T_{r+1}$ = $^nc_r(2)^{\frac{n-r}{4}}.\frac{1}{3^{r/4}}$ 5^{th} term from the beginning, put r = 4 $\Rightarrow T_5$ = $^nc_4(2)^{\frac{n-4}{4}}.\frac{1}{3}$ Now, 5^{th} term from the end = $(n-5+2)^{th}$ term from the beginning= $(n-3)^{rd}$ term For T_{n-3}, put r = n-4 $\Rightarrow T_{n-3}$ = $^nc_{n-4}(2)^{\frac{n-(n-4)}{4}}.\frac{1}{3^{\frac{n-4}{4}}}$ $\Rightarrow T_{n-3}$ = $^nc_{n-4}(2)^1.\frac{1}{3^{\frac{n-4}{4}}}$ Given, $\frac{T_5}{T_{n-3}}$ = $\frac{\sqrt{6}}{1}$ $\Rightarrow \frac{n_{C_4}(2)^{\frac{n-4}{4}}.\frac{1}{3}}{n_{C_{n-4}}(2)^1.\frac{1}{3^{\frac{n-4}{4}}}} = \frac{\sqrt{6}}{1}$ $\Rightarrow \frac{\frac{n!}{4!(n-4)!}.(2)^{\frac{n-4}{4}-1}.(3)^{\frac{n-4}{4}}}{\frac{n!}{(n-4)!4!}.(3)} = \frac{\sqrt{6}}{1}$ $\Rightarrow (2)^{\frac{n-8}{4}}.(3)^{\frac{n-4}{4}}$=$\sqrt{6}$ $\Rightarrow (6)^{\frac{n-8}{4}}$ = $(6)^{\frac{1}{2}}$ $\Rightarrow \frac{n-8}{4}$ = $\frac{1}{2}$ $\Rightarrow 2n - 16 = 4$ $\Rightarrow 2n = 20$ $\Rightarrow n = 10$ ans.	
Q.6)	Prove that there is no term including x^6 in the expansion of $\left(2x^2 - \frac{3x}{11}\right)^{11}$?	
Sol.6)	General terms: T_{r+1} = $(-1)^r 11_{C_r}(2x^2)^{11-r}\left(\frac{3}{x}\right)^r$ $\Rightarrow T_{r+1}$ = $(-1)^r 11_{C_r}(2)^{11-r}.(x)^{22-2r}.\frac{3}{x^r}^r$ $\Rightarrow T_{r+1}$ = $(-1)^r 11_{C_r}(2)^{11-r}.(3)^r.(x)^{22-3r}$	

	Let x^6 occurs in the $(r+1)^{th}$ $term$ then, for x^6 put $22-3r=6$ $\Rightarrow 3r=16$ $\Rightarrow r=\frac{16}{3}$, which is in fraction but 'r' cannot be in fraction or negative $\therefore$ there is no term in the expansion which involves x^6 ans.	
Q.7)	If the 4th term in the expansion of $\left(ax-\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then find the values of 'a' & 'n'?	
Sol.7)	Expansion: $\left(ax-\frac{1}{x}\right)^n$ General term: $T_{r+1}={}^nc_r(ax)^{n-r}\left(\frac{1}{x}\right)^r$ $\Rightarrow T_{r+1}={}^nc_r a^{n-r}.x^{n-r}.\frac{1}{x^r}$ $\Rightarrow T_{r+1}={}^nc_r a^{n-r}.x^{n-2r}$ For T_4, put $r=3$ $\Rightarrow T_4={}^nc_3 a^{n-3}.x^{n-6}$ Given that, $T_4=\frac{5}{2}$ $\therefore {}^nc_3 a^{n-3}.x^{n-6}=\frac{5}{2}$ .... (1) Clearly R.H.S, of above equation is independent of x $\therefore$ put $n-6=0$ $\Rightarrow n=6$, put $n=6$ in equation (1) $\Rightarrow {}^6c_3 a^3.x^0=\frac{5}{2}$ $\Rightarrow \frac{6\times5\times4}{6}.a^3=\frac{5}{2}$ $\Rightarrow a^3=\frac{5}{40}=\frac{1}{8}=\frac{1}{23}$ $\Rightarrow a^3=\left(\frac{1}{2}\right)^3 \Rightarrow a^3=\frac{1}{2}$ $\therefore n=6$ & $a=\frac{1}{2}$	
Q.8)	If a_1, a_2, a_3 and a_4 be the coefficient of four consecutive terms in the expansion of $(1+x)^n$, then show that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$?	
Q.8)	Expansion $(1+x)^n$ General term: $T_{r+1}={}^nc_r(1)^{n-r}(x)^r$ $T_{r+1}={}^nc_r x^r$ Let the four consecutive terms are $r^{th}, (r+1)^{th}, (r+2)^{th}$ and $(r+3)^{th}$ $T_r={}^nc_{r-1}.x^{r-1} \Rightarrow$ coefficient of ${}^nc_{r-1}=a_1$ $T_{r+1}={}^nc_r.x^r \Rightarrow$ coefficient of ${}^nc_r=a_2$ $T_{r+2}={}^nc_{r+1}.x^{r+1} \Rightarrow$ coefficient of ${}^nc_{r+1}=a_3$ $T_{r+3}={}^nc_{r+2}.x^{r+2} \Rightarrow$ coefficient of ${}^nc_{r+2}=a_4$ Now, $a_1+a_2={}^nc_{r-1}+{}^nc_r={}^{n+1}c_r$ $a_2+a_3={}^nc_r+{}^nc_{r+1}={}^{n+1}c_{r+1}$ $a_3+a_4={}^nc_{r+1}+{}^nc_{r+2}={}^{n+1}c_{r+2}$ ${}^nc_{r-1}+{}^nc_r={}^{n+1}c_r$ (property 1) taking L.H.S. $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$ $=\frac{{}^nc_{r-1}}{{}^{n+1}c_r}+\frac{{}^nc_{r+1}}{{}^{n+1}c_{r+2}}$ $=\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{(n+1)!}{r!(n+1-r)!}}+\frac{\frac{n!}{(r+1)!(n-r-1)!}}{\frac{(n+1)!}{(r+2)!(n+1-r-2)!}}$ $=\frac{n!r!}{(n+1)!(r-1)!}+\frac{n!(r+2)!}{(n+1)!(r+1)!}$ $=\frac{n!r(r-1)!}{(n+1)n!(r-1)!}+\frac{n!(r+2)(r+1)!}{(n+1)n!(r+1)!}$	

	$= \frac{r}{n+1} + \frac{r+2}{n+1}$ $= \frac{2r+2}{n+1} = \frac{2(r+1)}{n+1}$ …………… (1) Taking R.H.S. $\frac{2a_2}{a_2+a_3}$ Do yourself and get R.H.S = $\frac{2(r+1)}{n+1}$ ………………… (2) From eq. (1) and eq. (2), L.H.S. = R.H.S (proved)	