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BINOMIAL THEOREM

	QUESTIONS BASED ON "GENERAL TERMS"	
Q.1)	Find the positive value of 'm' for which the coefficient of x^2 in the expansion of $(1+x)^m$ is 6?	
mcqs	a) 6 b) 9 c) 4 d) 1	
Sol.1)	Given expansion: $(1+x)^m$ coefficient of x^2 = 6 To find : m General term: $T_{r+1} = {}^mc_r(1)^{m-r}x^r = T_{r+1} = {}^mc_r x^r$ For x^2 put r=2 $\therefore\ T_3 = {}^mc_2 x^r$ Here, coefficient of x^2 = 6 $\Rightarrow\ {}^mc_2 = 6$ $\Rightarrow\ \frac{m(m-1)}{2} = 6$ $\Rightarrow\ m^2$-m-12=0 $\Rightarrow (m-4)(m+3) = 0$ ⇒ m=4 or m= -3 but m cannot negative(-) ∴ m ≠ -3 ∴ m= 4 ans.	
Q.2)	If the coefficient of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in this expansion of $(1+x)^{34}$ are equal. Find the value of 'r'?	
mcqs	a) 14 b) 10 c) 12 d) 20	
Sol.2)	Given expansion: $(1+x)^{34}$ Coefficient of $T_{r-5} = T_{2r-1}$ To find 'r' General term: $T_{r+1} = {}^{34}c_r x^2 (1)^{34-r} x^r$ = $T_{r+1} = {}^{34}c_r x^2$ For T_{r-5} , put r = r-6 $\therefore T_{r-5} = {}^{34}c_{r-6}\, x^{r-6}$ Here coefficient of $T_{r-5} = {}^{34}c_{r-6}$ For T_{2r-1} , put r = 2r-2 $\therefore T_{2r-2} = {}^{34}c_{2r-2}\, x^{2r-2}$ Here coefficient of $T_{2r-1} = {}^{34}c_{2r-2}$ We are given that, coefficients are equal $\Rightarrow {}^{34}c_{r-6} = {}^{34}c_{2r-2}$ ⇒ r-6 + 2r-2 = 34..........(if ${}^nc_x = {}^nc_y$ then x+y=n or x=y) (Or) r-6 = 2r-2 ⇒ 3r=42 or r – 4 but 'r' cannot be negative(-) ⇒ r=14 ans.	
Q.3)	Find the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^6$?	
mcqs	a) $\frac{-9}{1}$ b) $\frac{5}{12}$ c) 10 d) $\frac{2}{8}$	
Sol.3)	General term is given by $T_{r+1} = (-1)^r \cdot {}^6c_r \frac{(3)^{6-2r}}{2^{6-r}} \cdot x^{12-3r}$ For independent term of x i.e. x^0 , put 12-3r = 0 ⇒ r = 4 $\therefore T_5 = (-1)^4 \cdot {}^6c_4 \frac{(3)^{6-8}}{2^2} \cdot x^0$ = ${}^6c_2 \frac{(3)^{-2}}{2^2}$........... $(6c_4 = 6c_2)$	

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	$= \frac{6\times5}{2} \times \frac{1}{9\times4} = \frac{5}{12}$ $\therefore$ 5th term is the independent term of x and is given by $\frac{5}{12}$ ans.
Q.4)	Find the value of 'a' so that the term independent of 'x' in $\left(\sqrt{x}+\frac{a}{x^2}\right)^{10}$ is 405?
Mcqs	a) $a^2 = \frac{40\times9}{8\times11}$ b) $a^2 = \frac{-405\times2}{9\times10}$ c) $a^2 = \frac{405\times2}{9\times10}$ d) $a^2 = \frac{205\times2}{3\times6}$
Sol.4)	Given expansion: $\left(x^{1/2}+\frac{a}{x^2}\right)^{10}$ Independent term of x = 405 To find 'a' General term: $T_{r+1} = {}^{10}c_r \left(x^{1/2}\right)^{10-r} . \frac{a^r}{x^{2r}}$ $\Rightarrow T_{r+1} = {}^{10}c_r\, (x)^{\frac{10-r}{2}-2r} . \frac{a^r}{x^{2r}}$ $\Rightarrow T_{r+1} = {}^{10}c_r\, (x)^{\frac{10-r}{2}-2r} . a^r$ $\Rightarrow T_{r+1} = {}^{10}c_r\, (x)^{\frac{10-5r}{2}} . a^r$ Now, for independent term of x i.e. x^0 , Put $\frac{10-5r}{2} = 0$ $\Rightarrow$ r = 2 $\therefore T_3 = {}^{10}c_2\, (x)^0 . a^2$ $T_3 = {}^{10}c_2\, a^2$ Also independent term of x = 405...........(given) $\Rightarrow {}^{10}c_2\, a^2 = 405$ $\Rightarrow \frac{10\times9}{2}\, a^2 = 405$ $\Rightarrow a^2 = \frac{405\times2}{9\times10}$ ans.
Q.5)	Find the middle terms in the expansion of $\left(3x-\frac{x^3}{6}\right)^7$?
Mcqs	e) $42x^{13}$ and 35 f) $\frac{-105}{8}x^{13}$ and $\frac{35}{48}x^{15}$ g) $\frac{25}{72}x^{13}$ and $\frac{30}{48}$ h) $\frac{-10}{1}x^1$ and $\frac{35}{8}x^5$
Sol.5)	Given expansion: $\left(3x-\frac{x^3}{6}\right)^7$ To find 'middle term' Since, power is odd, $\therefore$ there are two middle terms = $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ i.e. $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+3}{2}\right)^{th}$ $\Rightarrow$ 4th and 5th terms General term: $T_{r+1} = (-1)^r\, {}^7c_r\, (3x)^{7-r} \left(\frac{x^3}{6}\right)^2$ $= (-1)^r\, {}^7c_r\, (3)^{7-r} . x^{7-r} . \frac{x^{3r}}{6^r}$ $= T_{r+1} = (-1)^r\, {}^7c_r\, \frac{(3)^{7-r}}{6^r}\, x^{7+2r}$ For T_4 , put r = 3 $= T_3 = (-1)^r\, {}^7c_3\, \frac{(3)^4}{6^3}\, x^{7+6}$ $= \frac{-7\times6\times5}{6} \times \frac{81}{216}\, x^{13}$ $= T_4 = \frac{-105}{8} x^{13}$ For T_5 , put r = 4 $\therefore T_5 = (-1)^r\, {}^7c_4\, \frac{(3)^3}{6^4}\, x^{7+8}$

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	= $T_5 = \frac{35}{48}x^{15}$ $\therefore$ the middle terms are $\frac{-105}{8}x^{13}$ and $\frac{35}{48}x^{15}$ ans.	
Q.6)	Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.........(2n-1).2^n.x^n}{n!}$?	
Mcqs	a) False b) True c) Not proved d) Negative	
Sol.6)	Given expansion $(1+x)^{2n}$ Since, power $(2n)$ is even, only 1 middle term = $\left(\frac{2n}{2}+1\right)^{th}$ = $(n+1)^{th}$ terms General term: T_{r+1} = ${}^{2n}c_r x^r$ For T_{n+1} , put r = n = T_{n+1} = ${}^{2n}c_n x^n$ = $\frac{(2n)!}{n!n!}x^n$ = $\frac{1.2.3.4.5.6.........(2n-1).(2^n).x^n}{n!n!}$ = $\frac{[1.3.5.........(2n-1)]\ [2.4.6.........(2^n)x^n]}{n!n!}$ = $\frac{[1.3.5.........(2n-1)].2^n\ (1.2.3.........n).x^n}{n!n!}$ = $\frac{1.3.5.........(2n-1).2^n.n!.x^n}{n!n!}$ = T_{n+1} = $\frac{1.3.5.........(2n-1).2^n.x^n}{n!}$ ans.	
Q.7)	Show that the coefficient of the middle terms in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$? For mcqs..... is it true or false?	
Mcqs	e) True f) False g) Negative h) Positive	
Sol.7)	1st expansion : $(1+x)^{2n}$ Since, power $(2n)$ is even , only 1 middle term = $\left(\frac{2n}{2}+1\right)^{th}$ = $(n+1)^{th}$ term General term: T_{r+1} = ${}^{2n}c_r x^r$ For T_{n+1} , put r = n = T_{n+1} = ${}^{2n}c_n x^n$..............(coefficient = ${}^{2n}c_n$) 2nd expansion: $(1+x)^{2n-1}$ Since, power $(2n-1)$ is odd , only 2 middle terms = $\left(\frac{2n-1+1}{2}\right)^{th}$ = $\left(\frac{2n-1+3}{2}\right)^{th}$ term = n^{th} and $(n+1)^{th}$ terms General term: T_{r+1} = ${}^{2n-1}c_r x^r$ For T_n , put r = n-1 $\therefore T_n =$ ${}^{2n-1}c_{n-1}\ x^{n-1}$ Coefficient = ${}^{2n-1}c_{n-1}$ For T_{n+1} , put r = n $\therefore T_{n+1} =$ ${}^{2n-1}c_n\ x^n$ Coefficient = ${}^{2n-1}c_n$ Now, we have to prove that ${}^{2n}c_n$ = ${}^{2n-1}c_{n-1}$ + ${}^{2n-1}c_n$ R.H.S = ${}^{2n-1}c_{n-1}$ + ${}^{2n-1}c_n$ = ${}^{2n-1+1}c_n$ ....(${}^{n}c_r + {}^{n}c_{r-1} = {}^{n+1}c_r$) = ${}^{2n}c_n$ = L.H.S (proved)	
Q.8)	Prove that the coefficient of x^n is the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of$(1+x)^{2n-1}$?	

	For mcqs….. is it true or false?	
Mcqs	i) True j) False k) Negative l) Positive	
Sol.8)	1st expansion: $(1+x)^{2n}$ General term: T_{r+1} = ${}^{2n}c_r x^r$ For x^n , put r = n = T_{n+1} = ${}^{2n}c_n x^n$ Coefficient of x^n = ${}^{2n}c_n$ 2nd expansion: $(1+x)^{2n-1}$ General term: T_{r+1} = ${}^{2n-1}c_r x^r$ For x^n , put r = n = T_{n+1} = ${}^{2n-1}c_n x^n$ Coefficient of x^n = ${}^{2n-1}c_n$ Now, we have to prove that = ${}^{2n}c_n$ = 2(${}^{2n-1}c_n$) R.H.S = 2. ${}^{2n-1}c_n$ = $\frac{2(2n-1)!}{n!(n-1)!}$ ………….(1) L.H.S = ${}^{2n}c_n$ = $\frac{(2n)!}{n!n!}$ = $\frac{(2n)(2n-1)!}{n!n(n-1)!}$ = $\frac{2(2n-1)!}{n!(n-1)!}$ ……………..(2) From (1) & (2) , R.H.S = L.H.S (proved)	
Q.9)	The sum of the coefficients of the 1st three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^m$ is 559. Find the term containing x^3 in the expansion?	
mcqs	a) $2582x^3$ b) $-5940x^3$ c) 5900 d) $5940x^3$	
Sol.9)	Given expansion: $\left(x-\frac{3}{x^2}\right)^m$ To find 'm' General term: T_{r+1} = $(-1)^r\ {}^mc_r\ (x)^{m-r}\left(\frac{3}{x^2}\right)^2$ = $(-1)^r\ {}^mc_r\ (x)^{m-r}\ \frac{3^r}{x^{2r}}$ = T_{r+1} = $(-1)^r\ {}^mc_r\ (3)^r\ (x)^{m-3r}$ For T_1 , put r = 0 = T_1 = $(-1)^0\ {}^mc_0\ (3)^0\ (x)^m$ = T_1 = x^m ∴ coefficient of T_1 = 1 For T_2 , put r = 1 = T_2 = $(-1)^1\ {}^mc_1\ 3^1\ x^{m-3}$ = T_2 = -$(m)(3)\ x^{m-3}$ ∴ coefficient of T_2 = -3m For T_3 put r =2 = T_3 = $(-1)^2\ {}^mc_2\ 3^2\ x^{m-6}$ = T_3 = mc_2.9 x^{m-6} = T_3 = $\frac{9m(m-1)}{2}\ x^{m-6}$ ∴ coefficient of T_3 = $\frac{9m(m-1)}{2}$ We are given that, 1 - 3m + $\frac{9m(m-1)}{2}$ = 559 ⇒ 2 - 6m + $9m^2$ - 9m = 1118 ⇒ $9m^2$ – 15m – 116 = 0 ⇒ $3m^2$ - 5m – 372 = 0 (divide by 3)	

	a = 3, b = -5, c = -375 By quadratic formula, m = $\frac{5\pm\sqrt{25+(4)(3)(372)}}{2\times 3}$ m = $\frac{5\pm\sqrt{4489}}{6}$ m = $\frac{5\pm 67}{6}$ m = $\frac{5+67}{6}$, m = $\frac{5-67}{6}$ m = $\frac{72}{6}$, m = $\frac{-62}{6}$ m= -12 (since power (m) cannot –ve) ∴ general term becomes = T_{r+1} = $(-1)^r\ {}^{12}c_r\ (3)^r\ (x)^{12-3r}$ For x^3 , put r =3 ∴ $T_4 = (-1)^3\ {}^{12}c_3\ (3)^3\ x^3$ = $\frac{-12\times 11\times 10}{6} \times 27 \times x^3$ = -5940x^3 ans.
Q.10)	The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in ratio 1:7:42. Find the value of 'n'?
mcqs	a) 33 b) 26 c) 55 d) 78
Sol.10)	Given expansion: $(1+a)^n$ General term: $T_{r+1} = {}^n c_r a^r$ Let the three consecutive terms are $(r-1)^{th}$, $(r)^{th}$ and $(r+1)^{th}$ term For T_{r-1} , put r= r-2 ∴ $T_{r-1} = {}^n c_{r-2} a^{r-2}$ Coefficient of $T_{r-1} = {}^n c_{r-2}$ For T_r , put r= r-1 ∴ $T_r = {}^n c_{r-1} a^{r-1}$ Coefficient of $T_r = {}^n c_{r-1}$ $T_{r+1} = {}^n c_r a^r$ Coefficient of $T_{r+1} = {}^n c_r$ We are given that, ${}^n c_{r-2} : {}^n c_{r-1} : {}^n c_r$ = 1:7:42 consider, $\frac{{}^n c_{r-2}}{{}^n c_{r-1}} = \frac{1}{7}$ ⇒ $\frac{\frac{n!}{(r-2)!(n-r+2)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{1}{7}$ ⇒ $\frac{(r-1)!(n-r+1)!}{(r-2)!(n-r+2)!} = \frac{1}{7}$ ⇒ $\frac{(r-1)}{(n-r+2)!} = \frac{1}{7}$ ⇒ 7r-7 = n-r+2 ⇒ 8r-9 = n............(1) Now, consider $\frac{{}^n c_{r-1}}{{}^n c_r} = \frac{7}{42}$ ⇒ $\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{(r)!(n-r)!}} = \frac{1}{6}$ ⇒ $\frac{r!(n-r)!}{(r-1)!(n-r+1)!} = \frac{1}{6}$ ⇒ $\frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!} = \frac{1}{6}$ ⇒ $\frac{r}{n-r+1} = \frac{1}{6}$ ⇒ 6r + n – r +1 ⇒ 7r -1 = n.........(2)

	From (1) and (2), 8r-9 = 7r -1 ⇒ r=8, put in eq. (1) ⇒ n= 56 – 1 = 55 ans.	