

UNIT - 3 : STATISTICAL TOOLS AND INTERPRETATION

Ch-6 MEASURES OF DISPERSION

Points to rememeber

- * Dispersion is a measure of the variation of the items from central value.
- * The measures of dispersion are important to compare uniformity, consistency and reliability amongst variables/ senes
- * Absolute measures of dispersion are expressed in terms of original unit of series.
- * Relative measures are expressed in ratios or percentage, also known as coefficients of dispersion.

MEASURES OF DISPERSION

(i) Range	(ii) Inter quartile range	(iii) Quartile deviation or Semi Inter- quartile range
(iv) Mean deviation	(v) Standard Deviation	(vi) Lorenz curve

- * **Range :** Range is defined as the difference between two extreme observations i.e. the largest and the smallest value.

Symbolically

$$R = L - S$$

Where

R = Range

L = Largest Value

S = Smallest value

- * Coefficient of range = $\frac{L - S}{L + S}$

- * **Inter Quartile Range :**
Inter quartile range is the difference between upper quartile and lower quartile.

Inter-quartile range = $Q3 - Q1$

Where $Q3$ = Third quartile or upper quartile.

$Q1$ = First quartile or lower quartile

* Quartile Deviation :

Quartile deviation is known as half of difference of third quartile ($Q3$) and first quartile ($Q1$). It is also known as semi inter quartile range.

$$Q.D = \frac{Q3 - Q1}{2}$$

Where $Q.D$ = Quartile deviation

$Q3$ = Third quartile or upper quartile.

$Q1$ = First quartile of lower quartile.

Coefficient of quartile deviation

$$\text{Coefficient of } Q.D = \frac{Q3 - Q1}{Q3 + Q1}$$

Mean Deviation

Mean deviation / average deviation is the arithmetic mean of the deviations of various items from their average (mean, median or mode) generally from the median.

Calculation of mean deviation

Individual Series

Discrete Series

Continuous Series

$$M.D = \frac{\sum |D|}{N}$$

$$M.D = \frac{\sum f |D|}{N}$$

$$\frac{\sum f |D|}{N}$$

Where,

MD = Mean deviation

$|D|$ = Deviations from mean or median ignoring \pm Signs

N = Number of item (Individual Series)

N = Total number of Frequencies (Discrete and continuous series)

F = Number of frequencies.

Coefficient of mean deviation

$$\frac{\text{M.D.}}{\bar{X}} \quad \text{or} \quad \frac{\text{M.D.}}{M} \quad \text{or} \quad \frac{\text{M.D.}}{Z}$$

Standard Deviation :

Standard deviation is the best and widely used measure of dispersion. Standard deviation is the square root of the arithmetic mean of the squares of deviation of its items from their arithmetic mean.

Calculation of standard deviation in individual series.

Actual mean method.

$$\sigma = \frac{\sum X^2}{N}$$

Where σ = Standard Deviation
 X^2 = Square of deviation taken from mean
 N = Number of items

Shortcut method or assumed mean method

$$\sigma = \sqrt{\frac{\sum d X^2}{N} - \left[\frac{\sum d X}{N} \right]^2}$$

Where dx^2 = Square of deviation taken from assumed mean.

Calculation of standard deviation in discrete series :

Actual mean method or direct method

$$\sigma = \sqrt{\frac{\sum f X^2}{N}}$$

Where σ = S. D.
 $\sum fx^2$ = Sum total of the squared deviations
 Multiplied by frequency

N = Number of pair of observation.

Short cut method or assumed mean method

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left[\frac{\sum fd}{N} \right]^2}$$

Where σ = S. D.

$\sum fd^2$ = Sum total of the squared deviations

Multiplied by frequency

$\sum fd$ = Sum total of deviations multiplied by frequency.

N = Number of pair of observations.

Step deviation method

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left[\frac{\sum fd^1}{N} \right]^2} \times c$$

σ = Standard Deviation

$\sum fd^2$ = Sum total of the squared step deviations multiplied by frequency.

$\sum fd^1$ = Sum total of step deviations multiplied by frequency

C = Common factor

N = Number of pair of observation

Calculation of standard deviation in continuous series.

Actual mean method

$$\sigma = \sqrt{\frac{\sum fX^2}{N}}$$

σ = S.D.

$\sum fX^2$ = Sum total of the squared deviation multiplied by frequency.

N = Number of pair of observations.

Shortcut method or assumed mean method

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left[\frac{\sum fd}{N} \right]^2}$$

Step deviation method.

$$\sigma = \sqrt{\frac{\sum fd^{12}}{N} - \left[\frac{\sum fd^1}{N} \right]^2} \times c$$

Coefficient of variation

When two or more groups of similar data are to be compared with respect to stability (or uniformity or consistency or homogeneity), Coefficient of variation is the most appropriate measures.

$$C.V = \frac{\sigma}{\bar{X}} \times 100$$

Where C. V = Coefficient of variation

σ = Standard deviation

\bar{X} = Arithmetic mean

LORENZ CURVE :

- * The Lorenz curve devised by Dr. Max O. Lorenz, is a graphic method of studying dispersion.
- * The Lorenz curve always lies- below the line of equal distribution, unless the distribution is uniform.
- * The Area between the line of equal distribution and the plotted curve gives the extent of inequality in the items. The larger the area, more is the inequality.

ONE MARK QUESTIONS

1. What is inter quartile range?.
2. Give the formula of calculating coefficient of variation.
3. What is Lorenz Curve?
4. Calculate range
22, 35, 32, 45, 42, 48, 39
5. Which graphical method is used to measure dispersion?
6. Give the meaning of dispersion.
7. How is coefficient of mean deviation computed?
8. Which measure of dispersion covers middle 50% of the items?
9. Write one major demerit of mean deviation.
10. What do you mean by relative measure of dispersion?
11. What is a line of equal distribution.
12. Write two demerits of range.
13. Which is most widely used and best measurement of dispersion.
14. Give the formula of calculating quartile deviation.
15. Write two uses of range.

SHORT ANSWER TYPE QUESTIONS (3/4 MARKS)

1. Mention important measures of dispersion.
2. Mention any two merits and two demerits of mean deviation.
3. Distinguish between mean deviation and standard deviation.
4. What do you understand by dispersion?
Describe the various methods of computing dispersion.
5. Discuss the relative merits of range, mean deviation and standard deviation as measures of dispersion.
6. Find the range and coefficient of range of the following :

Marks :	10	20	30	40	50	60	70
No. of Students :	8	12	7	30	10	5	2

(Range = 60 marks : Coefficient of range = 0.75)

7. Find out the value of quartile deviation and its coefficient from the following data.

Roll No. :	1	2	3	4	5	6	7
Marks :	20	28	40	12	30	15	50

(Quartile deviation - 12.5 marks)

(Coefficient of quartile deviation = 0.45)

8. Calculate mean deviation from median and its coefficient from the following data :

100, 150, 80, 90, 160, 200, 140

(Mean deviation from median = 34.28)

(Coefficient of mean deviation = 0.74)

9. Calculate semi-interquartile range and its coefficient of the following data.

Marks :	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Std.	4	8	11	15	12	6	3

(Q. D = 11.55 Coefficient of Q.D = 0.337)

10. Calculate the standard deviation for the following data

5, 8, 7, 11, 14

(S. D = 3.16)

11. Coefficient of variation of two series are 58% and 69% and their standard deviation are 21.2 and 15.6 what are their means?

(Means \bar{X} = 36.55 and 22.60)

12. From the following data of two workers, identify who is more consistent worker?

	A	B
Average time in completing a job	40	42
Standard Eeviation	8	6

(Worker B is more consistent as his C.V.

(14.29%) is less than that of worker A (20%)

LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Discuss the merits, demerits and uses of range.
2. What is the meaning of Lorenz curve? State the steps involved in drawing a Lorenz curve.
3. What do you mean by mean deviation? In what way is mean deviation a better measure of dispersion than range and quartile deviation?
4. What do understand by dispersion? Describe the various methods of computing dispersion?.
5. Find the range and coefficient of range of the following:

Age in years :	5-10	10-15	15-20	20-25
Frequency :	10	15	20	5

(Range = 20 Coefficient of range = 0.67)

6. Find out quartile deviation, Interquartile range and coefficient of quartile deviation of the following series :

Height in inches:	58	59	60	61	62	63	64	65	66
No. of Plants:	2	3	6	15	10	15	4	3	1

(Q.D. = 1, Inter quartile range = 2 Coeff 4QD = 0.016)

7. Calculate mean deviation from median.

No. of fruits per plant :	0	1	2	3	4	5	6	7	8	9	10
No. of Plants :	2	5	7	11	18	24	12	8	6	4	3

(Me = 5, M.D = 1.68)

8. Find mean deviation from median of the marks secured by 100 students in a class test as given below :

Marks :	60-63	63-66	66-69	69-72	72-75
No. of Std.	5	18	42	27	8

(M. D. = 2.26)

9. Calculate coefficient of quartile deviation from the following data:

X (lessthan)	200	300	400	500	600
F	8	20	40	46	50

(Coefficient of quartile deviation 0.24)

10. Calculate standard deviation of the given data :

Size :	3	4	5	6	7	8	9
Frequency :	3	7	22	60	85	32	8

(S.D = 1.149)

11. Calculate standard deviation from the following series :

Class :	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency :	2	4	6	6	6	4	2

(S.D = 15.81)

12. The given table shows the daily income of workers of two factories.
Draw the Lorenz curves for both the factories.

Daily Income (Rs.) 0-100 100-200 200-300 300-400 400-500

Factory	A	8	7	5	3	2
Factory	B	15	5	2	1	1

13. The prices of share of company x and company y are given below.
State, which company is more stable?

Company X	25	50	45	30	70	42	36	48	34	60
Company Y	10	70	50	20	95	55	42	60	48	80

(C.V. of prices of share of x co. = 29.72%)

C.V. of prices of share of Y co = 45.94%

Prices of share of x co. is more stable.

14. Calculate coefficient of variation from the data given below :

X :	10	11	12	13	14	15	16
F :	10	25	15	5	15	20	10

($\bar{X} = 12.9$, S.D = 1.997, C.V. 15.5%)

15. Compare range, quartile deviation, mean deviation and standard deviation on the basis of calculations.
16. What is meant by mean deviation? Give the steps for calculating mean deviation in case of individual series.
17. Calculate the standard deviation from following data by step deviation method.

X :	0-10	10-20	20-30	30-40
F :	2	3	4	1

($\sigma = 9.165$)

ANSWERS OF VERY SHORT TYPE QUESTIONS (01 MARKS)

1. The difference in the two values of quartile is called inter quartile range (Q3 - Q1)
2. Coefficient of variation = $\frac{\sigma}{\bar{X}} \times 100$
3. Lorenz curve is the graphic presentation of studying dispersion.
4. Range = Largest value - Smallest value

$$= 48 - 22$$

$$= 26$$
5. Lorenz curve method is used to measure dispersion.
6. Dispersion is a measure of the variation of the item from a central value.
7. Mean deviation = $\frac{\sum f |D|}{N}$
8. Inter quartile range
9. The major demerit of mean deviation is that it ignores \pm signs.
10. Relative measures are expressed in ratios or percentage, also known as coefficients of dispersion.
11. While drawing Lorenz curve zero of X-axis and 100 on y-axis are joined by a line. This line is known as line of equal distributions.
12. Demerits of range
 - (i) It is not based on all the observation of series.
 - (ii) It is very much affected by extreme items.
13. The most widely used and best measurement of dispersion is standard deviation.
14. Quartile deviation = $\frac{Q3 - Q1}{2}$
15. Two uses of range -
 - (i) Quality control
 - (ii) Measure of fluctuations.