

INTRODUCTION TO TRIGONOMETRY

IMPORTANT CONCEPTS

TAKE A LOOK:

- Trigonometric ratios of an acute angle of a right angled triangle.

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Side Adjacent to } \angle \theta}{\text{Side Opposite to } \angle \theta} = \frac{AB}{BC}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta} = \frac{AC}{AB}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{side Opposite to } \angle \theta} = \frac{AC}{BC}$$

- Relationship between different trigonometric ratios

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

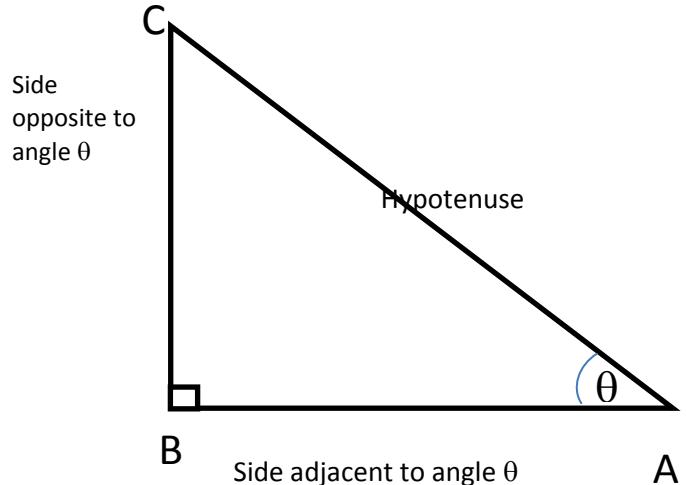
$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

- Trigonometric Identities.

- (i) $\sin^2 \theta + \cos^2 \theta = 1$
- (ii) $1 + \tan^2 \theta = \sec^2 \theta$
- (iii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

- Trigonometric Ratios of some specific angles.

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan \theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec \theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1



5. Trigonometric ratios of complementary angles.

- (i) $\sin(90^\circ - \theta) = \cos \theta$
- (ii) $\cos(90^\circ - \theta) = \sin \theta$
- (iii) $\tan(90^\circ - \theta) = \cot \theta$
- (iv) $\cot(90^\circ - \theta) = \tan \theta$
- (v) $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$
- (vi) $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

(Level – 1)

1. If θ and $30^\circ - 30^\circ$ are acute angles such that $\sin \theta = \cos(30^\circ - 30^\circ)$, then find the value of $\tan \theta$.

Ans. $\frac{1}{\sqrt{3}}$

2. Find the value of $\frac{(\cos 30^\circ + \sin 60^\circ)}{(1 + \cos 60^\circ + \sin 30^\circ)}$

Ans. $2\sqrt{3}$

3. Find the value of $(\sin \theta + \cos \theta)^2 + (\cos \theta - \sin \theta)^2$

Ans. 2

4. If $\tan \theta = \frac{3}{4}$ then find the value of $\cos^2 \theta - \sin^2 \theta$

Ans. $\frac{7}{25}$

5. If $\sec \theta + \tan \theta = p$, then find the value of $\sec \theta - \tan \theta$

Ans. $\frac{1}{p}$

6. Change $\sec^4 \theta - \sec^2 \theta$ in terms of $\tan \theta$.

Ans. $\tan^4 \theta + \tan^2 \theta$

7. If $\cot \theta = 1/\sqrt{3}$ then find the value of $(1 - \cos^2 \theta)/(1 + \cos^2 \theta)$

Ans. $\frac{3}{5}$

8. If $\cot \theta + \frac{1}{\cot \theta} = 2$ then find the value of $\cot^2 \theta + \frac{1}{\cot^2 \theta}$.

Ans. 2

9. If $\sin \theta = a/b$, then find the value of $\sec \theta + \tan \theta$

Ans. $\sqrt{\frac{b+a}{b-a}}$

10. If $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$, then find the value of x

Ans. 30°

11. If $0^\circ \leq x \leq 90^\circ$ and $2 \sin^2 x = 1/2$, then find the value of x

Ans. 30°

12. Find the value of $\operatorname{cosec}^2 30^\circ - \sin^2 45^\circ - \sec^2 60^\circ$

Ans. -2

13. Simplify $(\sec \theta + \tan \theta)(1 - \sin \theta)$

Ans. $\cos \theta$

Level - 2

1. If $\sec\alpha = 5/4$ then evaluate $\tan\alpha/(1+\tan^2\alpha)$. Ans: $\frac{12}{25}$
2. If $A+B = 90^\circ$, then prove that $\sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B}} - \frac{\sin^2 B}{\cos^2 B} = \tan A$
3. Prove that $\cos A/(1-\sin A) + \cos A/(1+\sin A) = 2\sec A$.
4. Prove that $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\cosec A$
5. Prove that $(\sin\theta + \cosec\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.
6. Evaluate $\frac{11\sin 70^\circ}{7\cos 20^\circ} - \frac{4\cos 53^\circ \cosec 37^\circ}{7\tan 15^\circ \tan 35^\circ \tan 55^\circ \tan 75^\circ}$ Ans: 1
7. Prove that $\sqrt{\frac{\cosec A - 1}{\cosec A + 1}} + \sqrt{\frac{\cosec A + 1}{\cosec A - 1}} = 2\sec A$.
8. In a right angle triangle ABC, right angled at B, if $\tan A = 1$, then verify that $2\sin A \cos A = 1$.
9. If $\tan(A-B) = \sqrt{3}$, and $\sin A = 1$, then find A and B. Ans: 90° & 30°
10. If θ is an acute angle and $\sin\theta = \cos\theta$, find the value of $3\tan^2\theta + 2\sin^2\theta - 1$. Ans: 3
11. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, prove that $x^2/a^2 + y^2/b^2 = 2$.

Level - 3

1. Evaluate the following :- $\sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ)$.
- Ans: 2
2. If $\frac{\cos\alpha}{\cos\beta} = m$, and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2+n^2)\cos^2\beta = n^2$.
3. Prove that $\tan^2\theta + \cot^2\theta + 2 = \cosec^2\theta \sec^2\theta$.
4. Prove that $(\tan A - \tan B)^2 + (1 + \tan A \tan B)^2 = \sec^2 A \sec^2 B$.
5. If $(\cos\theta - \sin\theta) = \sqrt{2} \sin\theta$, then show that $\cos\theta + \sin\theta = \sqrt{2} \cos\theta$.
6. Prove that $(\sin\theta + \sec\theta)^2 + (\cos\theta + \cosec\theta)^2 = (1 + \sec\theta \cosec\theta)^2$.

7. Prove that $\sin\theta/(1-\cos\theta) + \tan\theta/(1+\cos\theta) = \sec\theta\cosec\theta + \cot\theta$.

8. Prove that $(\sin\theta - \cosec\theta)(\cos\theta - \sec\theta) = \frac{1}{\tan\theta + \cot\theta}$.

9. If $\cot\theta = \frac{15}{8}$, evaluate $(2 + 2\sin\theta)(1 - \sin\theta)/(1 + \cos\theta)(2 - 2\sin\theta)$.

Level - 4

1. Prove that $(\sec\theta + \tan\theta - 1)/(\tan\theta - \sec\theta + 1) = \cos\theta/(1 - \sin\theta)$.

2. If $x = r \sin A \cos C$, $y = r \sin A \sin C$, $z = r \cos A$, Prove that $r^2 = x^2 + y^2 + z^2$.

3. Prove that $\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}$.

4. If $x = \sin\theta$, $y = \tan\theta$, prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

5. Prove that: $\frac{\cos\theta}{1 - \tan\theta} - \frac{\sin^2\theta}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta$

6. Evaluate $\frac{\sin^2\theta + \sin^2(90^\circ - \theta)}{3(\sec^2 61^\circ - \cot^2 29^\circ)} - \frac{3\cot^2 30^\circ \sin^2 54^\circ \sec^2 36^\circ}{2(\cosec^2 65^\circ - \tan^2 25^\circ)}$. Ans. $-\frac{25}{6}$

7. Prove that $\frac{1 + \cos A + \sin A}{1 + \cos A - \sin A} = \frac{1 + \sin A}{\cos A}$.

8. Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

9. Prove that $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$.

10. If $\cot\theta = \frac{7}{8}$, evaluate (i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$. Ans. 1, $-\frac{15}{113}$

Self Evaluation

1. If $a \cos\theta + b \sin\theta = c$, then prove that $a \sin\theta - b \cos\theta = \mp \sqrt{a^2 + b^2 - c^2}$.

2. If A, B, C are interior angles of triangle ABC, show that $\cosec^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1$.

3. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$.

4. If $\tan A = n \tan B$, $\sin A = m \sin B$, prove that $\cos^2 A = (m^2 - 1)/(n^2 - 1)$.

5. Evaluate $[\sec \theta \cosec(90^\circ - \theta) - \tan \theta \cot(90^\circ - \theta) + \sin^2 55^\circ \sin^2 35^\circ] /$

$(\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ)$. Ans: $\frac{2}{\sqrt{3}}$

6. If $\sec \theta + \tan \theta = p$, prove that $\sin \theta = (p^2 - 1)/(p^2 + 1)$.