

CHAPTER 5

TRIGONOMETRY

KEY POINTS

1. **Trigonometrical Ratios :** In $\triangle ABC$, $\angle B = 90^\circ$ for angle 'A'

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

2. **Reciprocal Relations :**

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}, \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}, \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} , \quad \cot \theta = \frac{1}{\tan \theta}$$

3. Quotient Relations :

$$\tan \theta = \frac{\sin \theta}{\cos \theta} , \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

4. Identities :

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta \Rightarrow \tan^2 \theta = \sec^2 \theta - 1 \text{ and } \sec^2 \theta - \tan^2 \theta = 1$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \Rightarrow \cot^2 \theta = \operatorname{cosec}^2 \theta - 1 \text{ and } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

5. Trigonometric Ratios of Some Specific Angles :

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

6. Trigonometric Ratios of Complementary Angles

$$\sin (90^\circ - \theta) = \cos \theta$$

$$\cos (90^\circ - \theta) = \sin \theta$$

$$\tan (90^\circ - \theta) = \cot \theta$$

$$\cot (90^\circ - \theta) = \tan \theta$$

$$\sec (90^\circ - \theta) = \operatorname{cosec} \theta$$

$$\operatorname{cosec} (90^\circ - \theta) = \sec \theta$$

MULTIPLE CHOICE QUESTIONS

Note : In the following questions $0^\circ \leq \theta \leq 90^\circ$

1. If $x = a \sin \theta$ and $y = a \cos \theta$ then the value of $x^2 + y^2$ is _____
(a) a (b) a^2
(c) 1 (d) $\frac{1}{a}$
2. The value of $\operatorname{cosec} 70^\circ - \sec 20^\circ$ is _____
(a) 0 (b) 1
(c) 70° (d) 20°
3. If $3 \sec \theta - 5 = 0$ then $\cot \theta =$ _____
(a) $\frac{5}{3}$ (b) $\frac{4}{5}$
(c) $\frac{3}{4}$ (d) $\frac{3}{5}$
4. If $\theta = 45^\circ$ then $\sec \theta \cot \theta - \operatorname{cosec} \theta \tan \theta$ is
(a) 0 (b) 1
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

5. If $\sin(90 - \theta) \cos \theta = 1$ and θ is an acute angle then $\theta =$ _____
- (a) 90° (b) 60°
(c) 30° (d) 0°
6. The value of $(1 + \cos \theta)(1 - \cos \theta) \operatorname{cosec}^2 \theta =$ _____
- (a) 0 (b) 1
(c) $\cos^2 \theta$ (d) $\sin^2 \theta$
7. $\triangle TRY$ is a right-angled isosceles triangle then $\cos T + \cos R + \cos Y$ is _____
- (a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $1 + \sqrt{2}$ (d) $1 + \frac{1}{\sqrt{2}}$
8. If $\sec \theta + \tan \theta = x$, then $\sec \theta =$
- (a) $\frac{x^2 + 1}{x}$ (b) $\frac{x^2 + 1}{2x}$
(c) $\frac{x^2 - 1}{2x}$ (d) $\frac{x^2 - 1}{x}$
9. The value of $\cot \theta - \sin\left(\frac{\pi}{2} - \theta\right) \cos\left(\frac{\pi}{2} - \theta\right)$ is _____
- (a) $\cot \theta \cos^2 \theta$ (b) $\cot^2 \theta$
(c) $\cos^2 \theta$ (d) $\tan^2 \theta$
10. If $\sin \theta - \cos \theta = 0$, $0 \leq \theta \leq 90^\circ$ then the value of θ is _____
- (a) $\cos \theta$ (b) 45°
(c) 90° (d) $\sin \theta$

11. $\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$ can be written as
- (a) $\cot \theta$ (b) $\sqrt{\sin \theta}$
 (c) $\frac{\sin \theta}{\sqrt{\cos \theta}}$ (d) $\tan \theta$
12. $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$ is equal to
- (a) $\sec^2 \theta + \tan^2 \theta$ (b) $\sec \theta - \tan \theta$
 (c) $\sec^2 \theta - \tan^2 \theta$ (d) $\sec \theta + \tan \theta$
13. In an isosceles right-angled ΔABC , $\angle B = 90^\circ$. The value of $2 \sin A \cos A$ is _____
- (a) 1 (b) $\frac{1}{2}$
 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$
14. If $\frac{\sin^2 20^\circ + \sin^2 70^\circ}{2(\cos^2 69^\circ + \cos^2 21^\circ)} = \frac{\sec 60^\circ}{K}$ then K is _____
- (a) 1 (b) 2
 (c) 3 (d) 4
15. If $\tan \theta = \frac{1}{\sqrt{7}}$, then $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} =$
- (a) $\frac{3}{4}$ (b) $\frac{5}{7}$
 (c) $\frac{3}{7}$ (d) $\frac{1}{12}$

SHORT ANSWER TYPE QUESTIONS

16. In $\triangle PQR$, $\angle Q = 90^\circ$ and $\sin R = \frac{3}{5}$, write the value of $\cos P$.
17. If A and B are acute angles and $\sin A = \cos B$ then write the value of $A + B$.
18. If $4 \cot \theta = 3$ then write the value of $\tan \theta + \cot \theta$.
19. Write the value of $\cot^2 30^\circ + \sec^2 45^\circ$.
20. Given that $16 \cot A = 12$, find the value of $\frac{\sin A + \cos A}{\sin A - \cos A}$.
21. If $\theta = 30^\circ$ then write the value of $\sin \theta + \cos^2 \theta$.
22. If $1 - \tan^2 \theta = \frac{2}{3}$ then what is the value of θ .
23. Find the value of θ of $\sqrt{3} \tan 2\theta - 3 = 0$.
24. If θ and ϕ are complementary angles then what is the value of $\operatorname{cosec} \theta \sec \phi - \cot \theta \tan \phi$
25. If $\tan(3x - 15^\circ) = 1$ then what is the value of x .
26. If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angles. Find the value of θ .

LONG ANSWER TYPE QUESTIONS

27. Simplify :
$$\tan^2 60^\circ + 4 \cos^2 45^\circ + 3 (\sec^2 30^\circ + \cos^2 90^\circ)$$
28. Evaluate
$$2 \left(\frac{\cos 58^\circ}{\sin 32^\circ} \right) - \sqrt{3} \left(\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ} \right)$$
29. Prove that
$$\operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta = \cot^2 \theta + \cot^4 \theta.$$

30. If $\sin \theta + \sin^2 \theta = 1$ then find the value of $\cos^2 \theta + \cos^4 \theta$.
31. If $\sin 2\theta = \cos(\theta - 36^\circ)$, 2θ and $\theta - 26^\circ$ are acute angles then find the value of θ .
32. If $\sin(3x + 2y) = 1$ and $\cos(3x - 2y) = \frac{\sqrt{3}}{2}$, where $0 \leq (3x + 2y) \leq 90^\circ$ then find the value of x and y .
33. If $\sin(A + B) = \sin A \cos B + \cos A \sin B$ then find the value of
 - (a) $\sin 75^\circ$
 - (b) $\cos 15^\circ$
34. Prove that $\frac{\cos A}{1 - \tan A} + \frac{\cos A}{1 - \cot A} = \cos A$, $A \neq 45^\circ$.
35. Prove that $\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta$
36. Find the value of

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ$$
37. Prove that

$$\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\cos \theta}{1 - \sin \theta}.$$
38. If $2 \sin(3x - 15) = \sqrt{3}$ then find the value of

$$\sin^2(2x + 10) + \tan^2(x + 5).$$
39. Find the value of $\sin 60^\circ$ geometrically.
40. Let $p = \tan \theta + \sec \theta$ then find the value of $p + \frac{1}{p}$.
41. Find the value of

$$\frac{-\tan \theta \cot(90^\circ - \theta) + \sec \theta \operatorname{cosec}(90^\circ - \theta) + \sin^2 35^\circ + \sin^2 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 30^\circ \tan 70^\circ \tan 80^\circ}$$

42. If $\frac{\cos \alpha}{\cos \beta} = m$ and $\frac{\cos \alpha}{\sin \beta} = n$ show that $(m^2 + n^2) \cos^2 \beta = n^2$.

43. Prove that $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ = 0$.

44. Prove that $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{2 \sec^2 \theta}{\tan^2 \theta - 1}$.

45. If A, B, C are the interior angles of a triangle ABC , show that

$$\sin\left(\frac{B+C}{2}\right) \cos\frac{A}{2} + \cos\left(\frac{B+C}{2}\right) \sin\frac{A}{2} = 1.$$

ANSWERS

1. b

2. a

3. c

4. a

5. d

6. b

7. a

8. b

9. a

10. b

11. d

12. d

13. a

14. d

15. a

16. $\cos P = \frac{3}{5}$

17. 90°

18. $\frac{25}{12}$

19. 5

20. 7

21. $\frac{5}{4}$

22. 30°

23. 30°

24. 1

25. $x = 20$.

26. 10°

27. 9

28. 1

30. 1

31. 42°

32. $x = 20, y = 15$

33. $\frac{\sqrt{3} + 1}{2\sqrt{2}}, \frac{\sqrt{3} + 1}{2\sqrt{2}}$, take $A = 45^\circ, B = 30^\circ$

34. –

35. –

36. $\frac{17}{2}$

37. –

38. $\frac{13}{12}$

39. –

40. $2 \sec \theta$

41. $2\sqrt{3}$

42. –

43. 60°

44. $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$