

## INTRODUCTION TO TRIGONOMETRY

## IMPORTANT CONCEPTS

## TAKE A LOOK:

1. Trigonometric ratios of an acute angle of a right angle triangle.

$$\sin \theta = \frac{\text{Side opposite to } (\theta)}{\text{Hypotenuse}} = \frac{BC}{AC}$$

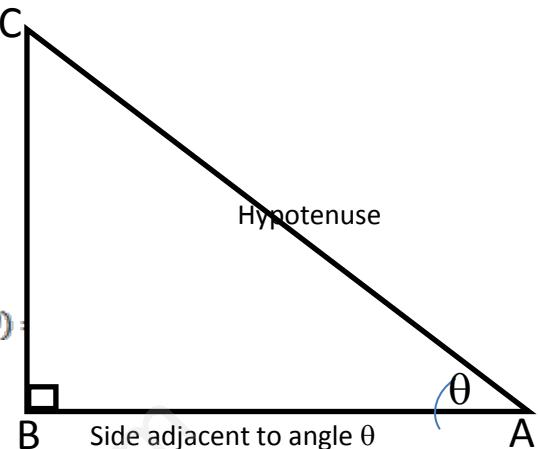
$$\cos \theta = \frac{\text{Side adjacent to } (\theta)}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan \theta = \frac{\text{Side opposite to } (\theta)}{\text{Side adjacent to } (\theta)} = \frac{BC}{AB}$$

$$\cot(\theta) = 1/(\tan \theta) = (\text{Side Adjacent to } (\theta)) / (\text{Side Opposite to } (\theta))$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to } (\theta)} = \frac{AC}{AB}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{side Opposite to } (\theta)} = \frac{AC}{BC}$$



2. Relationship between different trigonometric ratios

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

3. Trigonometric Identities.

- (i)  $\sin^2 \theta + \cos^2 \theta = 1$
- (ii)  $1 + \tan^2 \theta = \sec^2 \theta$
- (iii)  $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

4. Trigonometric Ratios of some specific angles.

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan \theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec \theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

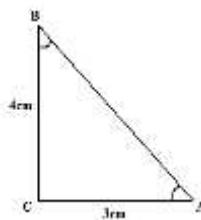
5. Trigonometric ratios of complementary angles.

- (i)  $\sin (90^\circ - \theta) = \cos \theta$
- (ii)  $\cos (90^\circ - \theta) = \sin \theta$
- (iii)  $\tan (90^\circ - \theta) = \cot \theta$
- (iv)  $\cot (90^\circ - \theta) = \tan \theta$

- (v)  $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$   
 (vi)  $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

#### LEVEL – 1

Questions carrying 1 marks



1. In the adjoining figure find the values of (a)  $\sin A$  (b)  $\cos B$
2.  $\sin 2A = 2 \sin A$  is true when  $A = \underline{\hspace{2cm}}$ .
3. Evaluate  $\sin 60^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 60^\circ$ .
4. If  $\tan A = \cot B$ , then show that  $A+B = 90^\circ$ .
5. Evaluate  $\sin^2 35^\circ + \sin^2 55^\circ$ .

Ans : (1)a.  $4/5$  b. $4/5$  (2)  $A=0^\circ$  (3) 1 (5) 1

#### LEVEL – 2

Questions carrying 2 marks

1. Evaluate  $3\sin^2 45^\circ + 2\cos^2 30^\circ - \cot^2 30^\circ$
2. Evaluate  $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$ .
3. If  $\tan 2A = \cot(A - 18^\circ)$ . Find the value of A.

4. If A, B and C are interior angles of  $\triangle ABC$  then show that  $\sin \frac{B+C}{2} = \cos \frac{A}{2}$
5. Prove that  $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Ans : (1) 0 (2)  $\sqrt{3}$  (3)  $A=36^\circ$

#### LEVEL – 3

Questions Carrying 3 marks

1. Express the trigonometric ratios of  $\sin A$ ,  $\sec A$  and  $\tan A$  in terms of  $\cot A$ .
2. Prove that  $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta) = 2$
3. Prove that  $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \tan A + \cot A$

4. If  $16 \cot A = 12$  Then find the value of  $\frac{\sin A + \cos A}{\sin A - \cos A}$
5. Prove that  $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdots \tan 89^\circ = 1$

#### LEVEL – 4

Questions Carrying 4 marks

1. Prove that  $\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\cos \theta}{1 - \sin \theta}$
2. If  $\sec \theta + \tan \theta = P$ , prove that  $\sin \theta = \frac{P^2 - 1}{P^2 + 1}$
3. If  $\tan \theta + \sin \theta = m$  and  $\tan \theta - \sin \theta = n$  show that  $m^2 - n^2 = 4\sqrt{mn}$
4. Prove the following  $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$
5. Prove that  $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

#### SELF EVALUATION

1. If  $\tan(A+B) = \sqrt{3}$  and  $\tan(A - B) = \frac{1}{\sqrt{3}}$  then find A and B. [  $0^\circ < A+B \leq 90^\circ$ ,  $A > B$  ]
2. If  $\tan A = \sqrt{3}$  Find other trigonometric ratios of  $\angle A$ .

3. If  $7\sin^2\theta + 3\cos^2\theta = 4$  show that  $\tan\theta = \frac{1}{\sqrt{3}}$
4. Find the value of  $\sin 60^\circ$  geometrically.
5. Evaluate  $\frac{2\sin^2 63^\circ + 1 + 2\sin^2 27^\circ}{3\cos^2 17^\circ - 2 + 3\cos^2 73^\circ}$
6. Evaluate:  $(\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3} (\tan 5^\circ \cdot \tan 15^\circ \cdot \tan 30^\circ \cdot \tan 75^\circ \cdot \tan 85^\circ)$
7. Prove that  $\sqrt{\frac{1+\cos A}{1-\cos A}} = \cosec A + \cot A$
8. Prove that:  $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left[ \frac{1-\tan\theta}{1-\cot\theta} \right]^2 = \tan^2\theta$

## STATISTICS

### IMPORTANT CONCEPTS

### TAKE A LOOK

The three measures of central tendency are :