

## **INTRODUCTION TO TRIGONOMETRY**

### **Level 1 (1 Mark)**

1.What is the Value of  $\sin^2 A + \cos^2 A$ ?

Sol. $\sin^2 A + \cos^2 A = 1$

2.What is the Value of  $\tan(90^\circ - A)$  ?

Sol. $\cot A$

3. If  $\tan A = \cot B$ , What is the Value of  $A+B$  ?

Sol. $\tan A = \cot A$

$\tan A = \tan A (90^\circ - B)$

$A = 90^\circ - B$

$A + B = 90^\circ$  Ans

4.What is the Value of  $\sec 30^\circ$  ?

Sol  $\frac{2}{\sqrt{3}}$

### **Level 2 (2 Marks)**

1. Evaluate  $\cos 60^\circ \sin 30^\circ + \sin 60^\circ + \cos 30^\circ$

Solution:  $\cos 60^\circ \sin 30^\circ + \sin 60^\circ + \cos 30^\circ$

$= 1/2 \times 1/2 + \sqrt{3}/2 \times \sqrt{3}/2$

$= 1/4 + 3/4 = 1+3/4 = 4/4 = 1$  Ans

2. If  $\sec^2 A (1 + \sin A)(1 - \sin A) = k$ , find the value of k.

Sol:  $\sec^2 A (1 - \sin^2 A) = k$

$\sec^2 A (\cos^2 A) = k$

$(1/\cos^2 A) \cos^2 A = k$

$k = 1$  Ans

3. If  $\sin A = 1/3$ , then find the value of  $(2 \cot^2 A + 2)$ .

Sol :  $2 \cot^2 A + 2 = 2(\cot^2 A + 1)$

$$= 2 (\operatorname{cosec}^2 A)$$

$$= 2 (1/\sin^2 A)$$

$$= 2/(1/3)^2 = 2/(1/9) = 2 \times 9/1 = 18 \text{ Ans}$$

4. 3 If  $\tan A = \cot B$ , prove that  $A + B = 90^\circ$

Sol:  $\tan A = \cot B$

$$\tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ \text{ Ans}$$

### Level 3 (3 Marks)

1. If  $\cot A = 7/8$  then what is the value of  $(1 + \cos A)(1 - \cos A)/(1 - \sin A)(1 + \sin A)$  ?

Sol:  $(1 + \cos A)(1 - \cos A)/(1 - \sin A)(1 + \sin A)$

$$\sin^2 A / \cos^2 A = \tan^2 A = 1 / \cot^2 A$$

$$\tan^2 A = 1 / \cot^2 A = 1 / (7/8)^2 = (8/7)^2 = 64/49 \text{ Ans}$$

2. Write the value of  $2 \cos^2 A + 2 / 1 + \cot^2 A$

Sol :  $2 \cos^2 A + 2 / 1 + \cot^2 A$

$$2 \cos^2 A + 2 / \operatorname{cosec}^2 A$$

$$2 \cos^2 A + 2 \sin^2 A$$

$$2(\cos^2 A + \sin^2 A)$$

$$2(1) = 2 \text{ Ans}$$

3. Given that  $\tan A = \frac{12}{5}$ , calculate  $\sin A$ ,  $\cos A$  and  $\sec A$ .

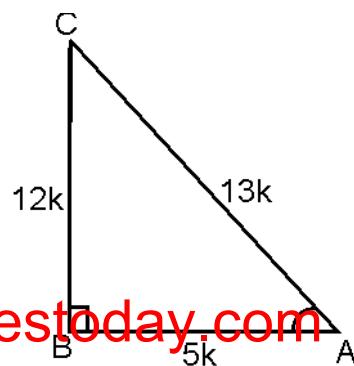
Sol : Let ABC be a triangle right angled at B.

As  $\tan A = \frac{12}{5}$ ,

Let  $BC = 12k$ ,

$AB = 5k$ .

Using Pythagoras Theorem,  $AC^2 = CB^2 + BA^2$



$$= (12k)^2 + (5k)^2 = 169k^2$$

$$\text{so, } AC = 13k$$

$$\sin A = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\cos A = \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\tan A = \frac{1}{\cos A} = \frac{13k}{5}$$

$$4. \sqrt{\frac{(1+\sin A)}{(1-\sin A)}} = \sec A + \tan A$$

$$\begin{aligned} \text{Sol: L.H.S} &= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}} \\ &= \frac{\sqrt{(1+\sin A)^2}}{\sqrt{1-\sin^2 A}} \\ &= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\ &= \sec A + \tan A = \text{R.H.S} \end{aligned}$$

#### **Level 4 (4 Marks)**

$$1. \frac{\cos^2 \theta}{1-\tan \theta} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} = 1 + \sin \theta \cos \theta$$

$$\begin{aligned} \text{Sol: L.H.S} &= \frac{\cos^2 \theta}{1-\frac{\sin \theta}{\cos \theta}} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} = \frac{\cos^3 \theta}{\cos \theta - \sin \theta} - \frac{\sin^3 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta \sin \theta} = \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \sin^2 \theta + \sin \theta \cos \theta)}{\cos \theta - \sin \theta} \\ &= 1 + \sin \theta \cos \theta \end{aligned}$$

2.. Prove that

$$\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$$

$$\text{Sol: L.H.S} = \frac{\tan \theta - \sec \theta - 1}{\tan \theta - \sec \theta + 1}$$

$$\begin{aligned}
 &= \frac{(\tan \theta - \sec \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1} \\
 &= \frac{1 + \sec \theta}{\cos \theta} = \text{R.H.S}
 \end{aligned}$$

3.: Prove that  $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2\cosec\theta$ .

$$\begin{aligned}
 \text{Sol: L.H.S} &= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}} \\
 &= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} = \frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\sin\theta} \\
 &= \frac{1+\cos\theta+1-\cos\theta}{\sin\theta} \\
 &= \frac{2}{\sin\theta} = 2\cosec\theta
 \end{aligned}
 \quad (\text{using } 1 - \cos^2\theta = \sin^2\theta)$$

4. If  $\sec\theta + \tan\theta = p$ , prove that  $\sin\theta = \frac{p^2-1}{p^2+1}$

Sol:  $\sec\theta + \tan\theta = p$ .....(i),

$$\sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1 \quad \text{.....(ii)}$$

Dividing (ii) by (i) we get

$$(\sec\theta - \tan\theta) = \frac{1}{p} \quad \text{.....(iii)}$$

Adding (i) and (iii) we get

$$\sec\theta - \tan\theta + \sec\theta + \tan\theta = p + \frac{1}{p}$$

$$\Rightarrow 2\sec\theta = \frac{1+p^2}{p} \quad \text{.....(iv)}$$

$$\text{Similarly, } 2\tan\theta = \frac{p^2-1}{p} \quad \text{.....(v)}$$

Dividing (v) by (iv) we get

$$\sin\theta = \frac{p^2-1}{p^2+1}$$