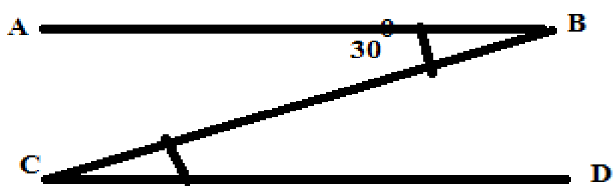
QUESTIONS OF HEIGHTS AND DISTANCES

CLASS X

LEVEL–1

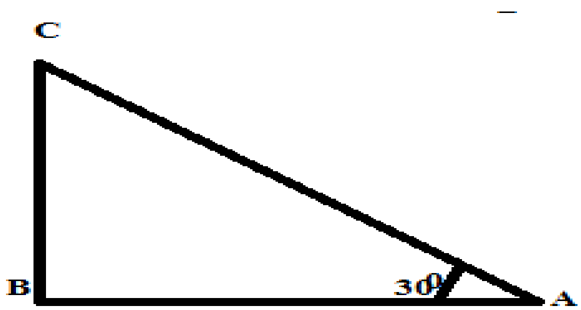
Q1. If the observer is at point B and the object is at point C. What is the angle formed? (Elevation /depression)



Solution. As the observer is at B and the object is at C. So the observer has to look downwards.

So the angle formed is the angle of depression.

Q2. In the given fig: find BC if AC = 6cm and $<A=30^0$.



Solution. In the given fig:

$<A=30^0$

or $\sin A=\frac{BC}{AC}$

or $\sin 30^0=\frac{BC}{6}$

or $\frac{1}{2}=\frac{BC}{6}$

or $BC = \frac{6}{2}=3cm$

Q3. A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60^0. Find the height of the tower.

Solution.

Let us assume the AB be the tower and C is a point 20 m away from the ground.

Angle of elevation of the top of the tower is 60°

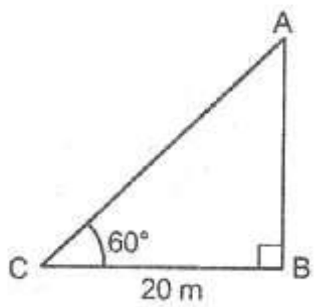
In rt.ΔCBA,

$$\frac{AB}{CB} = \tan 60^\circ$$

$$\frac{AB}{20} = \sqrt{3}$$

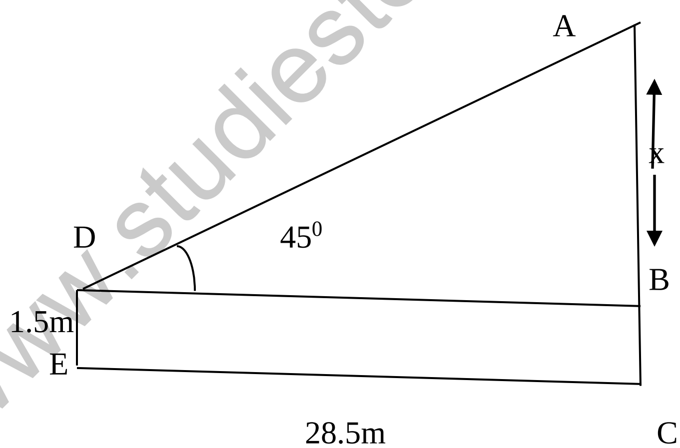
AB = $20\sqrt{3}$ m

Hence, the height of the tower is $20\sqrt{3}$ m.



Q4. An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the Chimney from his eye is 45^0. What is height of the chimney?

Solution.



Let the height of the chimney = x + 1.5

In ΔADB

Tan 45^0 = x / DB

1 = x / 28.5 { EC = DB}

X = 28.5cm

Therefore height of the chimney = 28.5 + 1.5

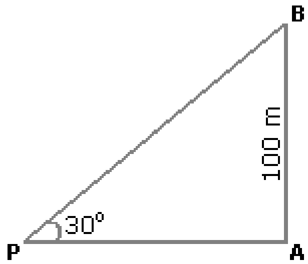
= 30 m(Ans)

LEVEL – 2

Q1. From a point P on a level ground, the angle of elevation of the top tower is 30º. If the tower is 100 m high, find the distance of point P from the foot of the tower.

Solution.

Let AB be the tower.



Then, ∠APB = 30º and AB = 100 m.

$\frac{AB}{AP} = \tan 30^\circ = \frac{1}{\sqrt{3}}$
$\Rightarrow$

AP = (AB x√3) m

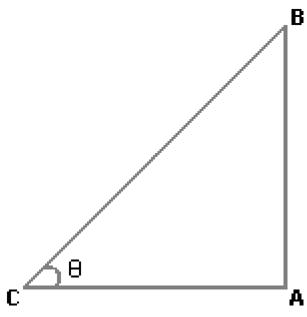
= 100√3 m

= (100 x 1.73) m

= 173m

Q2. Find the angle of elevation of the sun, when the length of the shadow of a tree √3 times the height of the tree.
Solution.

Let AB be the tree and AC be its shadow.



Let ∠ACB = θ.

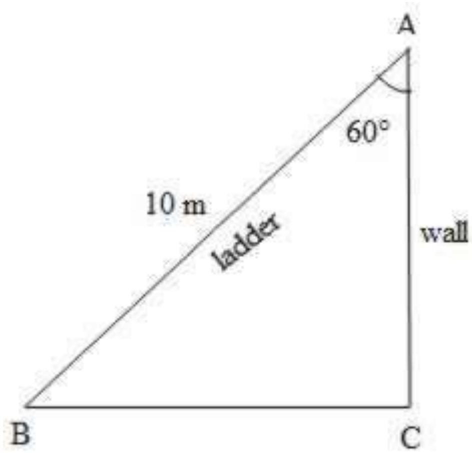
As, AC = √3 AB

Then, $\frac{AC}{AB} = \sqrt{3} \Rightarrow \cot\theta = \sqrt{3}$

∴ θ = 30º.

Q3. A ladder 10 m long just reaches the top of a wall and makes an angle of 60° with the wall.Find the distance of the foot of the ladder from the wall (√3=1.73)

Solution.



Let BA be the ladder and AC be the wall as shown above.

Then the distance of the foot of the ladder from the wall = BC

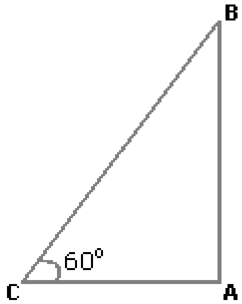
Given that BA = 10 m , ∠BAC = 60°

sin 60°=BC/BA $\sqrt{3}/2$=BC/10 BC = 10×$\sqrt{3}/2$=5×1.73=8.65 m

Q4. The angle of elevation of a ladder leaning against a wall is 60º and the foot of the ladder is 4.6 m away from the wall. Find the length of the ladder.

Solution.

Let AB be the wall and BC be the ladder.



Then, ∠ACB = 60° and AC = 4.6 m.

$\frac{AC}{BC} = cos60^0 = \frac{1}{2}$

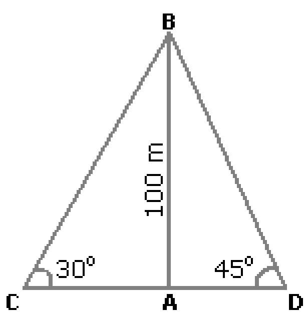
Or BC = 2 (AC)
Or BC = (2 × 4.6)m
Or BC = 9.2 m

LEVEL-3

Q1. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships is 30° and 45° respectively. If the lighthouse is 100 m high, find the distance between the two ships.

Solution.

Let AB be the lighthouse and C and D be the positions of the ships.



Then, AB = 100 m, ∠ACB = 30° and ∠ADB = 45°.

$\frac{AB}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$ ⇒ AC = AB x √3= 100√3

$\frac{AB}{AD} = \tan 45^\circ = 1$ ⇒ AD = AB = 100 m.

∴ CD = (AC + AD) = $(100\sqrt{3} + 100)$ m

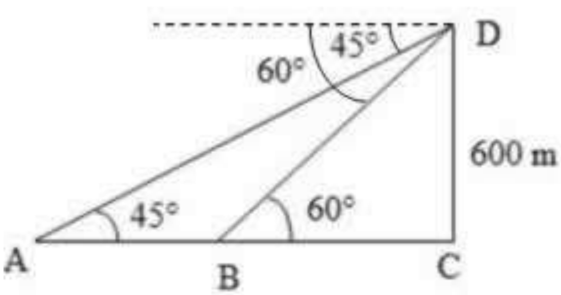
= $100(\sqrt{3}+ 1)$

= (100 x 2.73) m

= 273 m.

Q2. On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 600 m, find the distance between the objects.

Solution.



Let DC be the tower and A and B be the objects as shown above.

Given that DC = 600 m , ∠DAC = 45°, ∠DBC = 60°

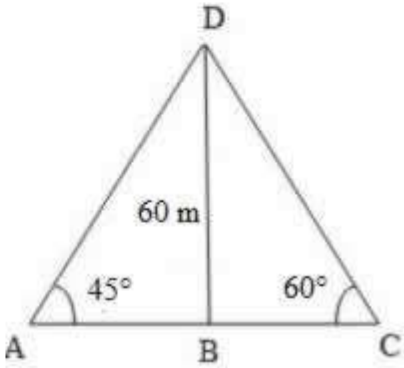
tan 60°=DC/BC $\sqrt{3}$=600/BC   BC= 600/$\sqrt{3}$------ (1)

tan 45°=DC/AC   1=600/AC   AC= 600------ (2)

Distance between the objects = AB = (AC - BC)=600−600$\sqrt{3}$ = 254 m(approx..)

Q3. The angle of elevation of the top of a lighthouse 60 m high, from two points on the ground on its opposite sides are 45° and 60°. What is the distance between these two points?

Solution.



Let BD be the lighthouse and A and C be the two points on ground.

Then, BD, the height of the lighthouse = 60 m

$\angle$BAD = 45° , $\angle$BCD = 60°

tan 45° = BD/BA ⇒1=60/BA⇒BA=60 m------ (1)

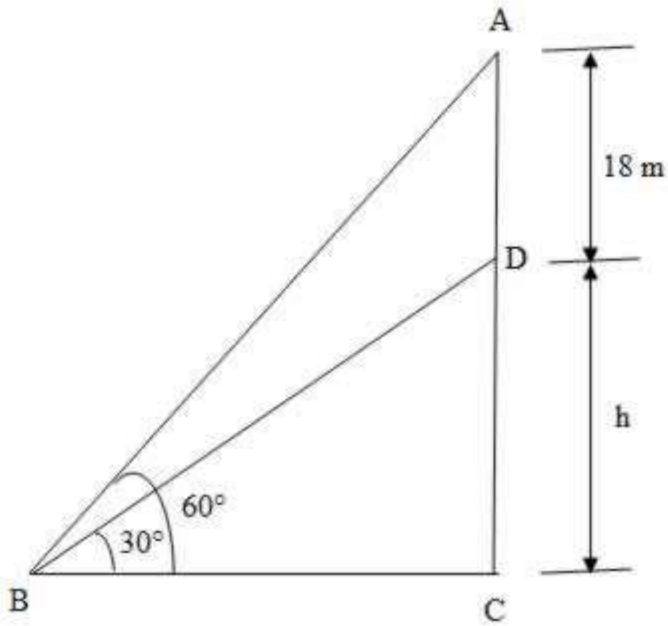
tan 60° = BD/BC ⇒$\sqrt{3}$=60/BC

⇒BC=60/$\sqrt{3}$=20×1.73=34.6 m------ (2)

Distance between the two points A and C = AC = BA + BC
= 60 + 34.6 [∵ Substituted the value of BA and BC from (1) and (2)]
= 94.6 m

Q4. 10. A vertical tower stands on ground and is surmounted by a vertical flagpole of height 18 m. At a point on the ground, the angle of elevation of the bottom and the top of the flagpole are 30° and 60° respectively. What is the height of the tower?

Solution.



Let DC be the vertical tower and AD be the vertical flagpole. Let B be the point of observation.

Given that AD = 18 m, ∠ABC = 60°, ∠DBC = 30°

Let DC be h.

tan 30°=DC/BC $1/\sqrt{3}$=h/BC   h=BC/√3------ (1)

tan 60°=AC/BC   √3=(18+h)/BC   18+h=BC×√3------ (2)

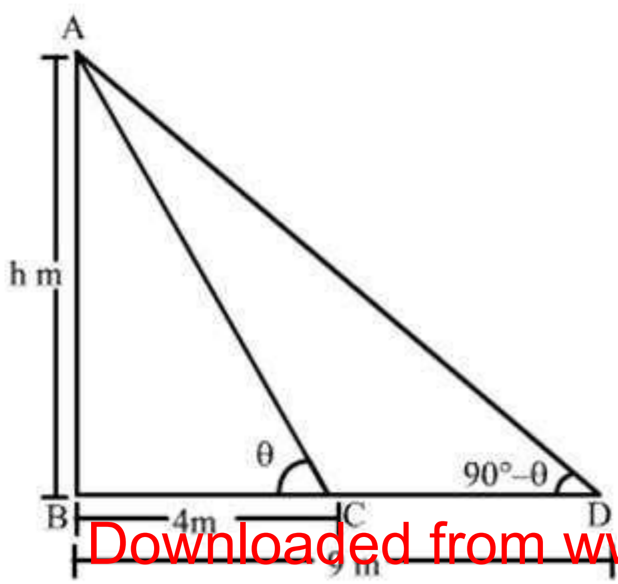
from (1) & (2) we get
the height of the tower 'h' = 9 m

LEVEL-4

1. **Q1.** The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution.

Let AB (= h metres) be the tower, and let C and D be two points of observation at distances 4 m and 9 m, respectively, from base B of the tower.



$\therefore$ BC = 4 m, BD = 9 m

$\angle ABC = \angle ABD = 90°$

Let $\angle ACB = \theta$.

$\therefore \angle ADB = (90° - \theta)$

In right triangle ABC, we have

$$tan\theta = \frac{AB}{BC}$$

$$\Rightarrow tan\theta = \frac{h}{4} \ldots (1)$$

In right triangle ABD, we have $\tan(90° - \theta) = \frac{AB}{BD}$

$$\Rightarrow cot\theta = \frac{h}{9} \ldots (2)$$

From equations (1) and (2), we get

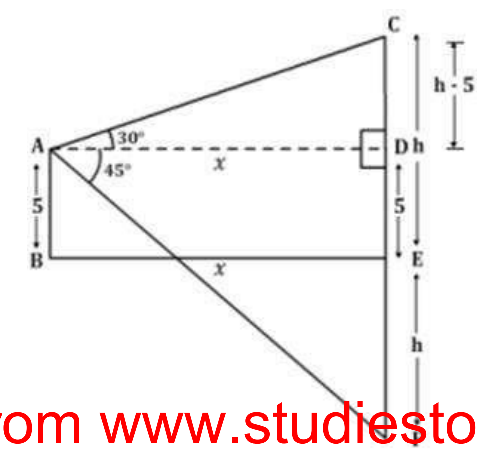
$$tan\theta \times cot\theta = \frac{h}{4} \times \frac{h}{9}$$

$$\Rightarrow 1 = \frac{h^2}{36}$$

$\Rightarrow 36 = h^2$
$\Rightarrow h = 6$

Hence, the height of the tower is 6 m.

Q2. The angles of elevation and depression, of an aeroplane and its image in water are 30° and 45°, respectively, from the top of a tree of height 5 m. Find the height at which the aeroplane is flying from the ground.
Solution.



Let AB be the height of the tree.
$\Rightarrow$ AB = 5 m

Let C and F be the actual position of the aeroplane and its image in the water, respectively.
From the figure: Height at which aeroplane is flying from ground = CE

⇒ CE = h

Distance of the aeroplane's image in water from ground = EF

⇒ EF = h

Given ∠CAD = 30° and ∠DAF = 45°

As per the problem, ADEB is a rectangle.

⇒ AB = DE = 5

Let AD = BE = x

In Δ DAF:

$$= \frac{DF}{AD}$$

tan 45°

$$\Rightarrow 1 = \frac{DE + EF}{AD}$$

$$\Rightarrow 1 = \frac{5 + h}{x}$$

$$\Rightarrow x = 5 + h$$

- - - - (1)

In Δ CAD:

$$tan30° = \frac{CD}{AD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CE - DE}{AD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-5}{x}$$

$$\Rightarrow x = \sqrt{3}(h-5)$$

- - - - (2)

Equating equations (1) and (2), we get

$$5 + h + \sqrt{3}(h-5)$$

$$\Rightarrow h = \frac{5(\sqrt{3}+1)}{(\sqrt{3}-1)}$$

On rationalising the denominator, we get

$$h = \frac{5(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{5\left(\sqrt{3}+1\right)^2}{3-1}$$

$$= \frac{5(3+1+2\sqrt{3})}{2}$$

$$= \frac{5(4+2\sqrt{3})}{2}$$

$$= 5(2+\sqrt{3})$$

Hence, the height of the aeroplane from the ground is $5(2+\sqrt{3})$m

Q3. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°. The car is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point

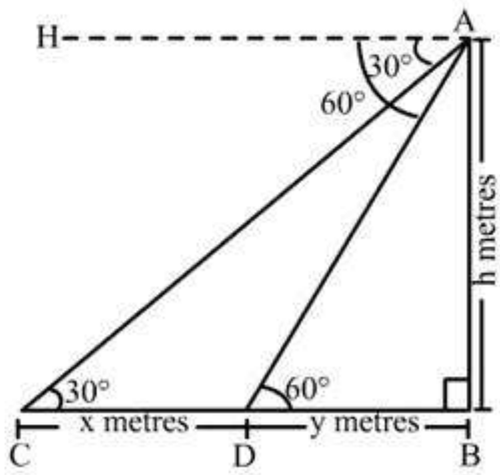
Solution.

Let AB (= h metres) be the tower.
Let C be the initial position of the car on the highway.

Let D be the position of the car after six seconds.

Let AH be the horizontal through A.

HA is parallel to CDB.



The angle of depression of point C at A is 30°, and the angle of depression of point D at A is 60°.

i.e. ∠ACB = ∠HAC = 30°

∠ADB = ∠HAD = 60°

∠ABC = ∠ABD = 90°

Let CD = x metres and let DB = y metres (distance to be covered to reach the tower).

In right-angled triangle ABD, we have

$$tan60° = \frac{AB}{DB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow \sqrt{3}y = h \quad \dots (1)$$

In right-angled triangle ABC, we have

$$tan30° = \frac{AB}{CB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{CD+DB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}y}{x+y}$$ [Using equation (1)]

⇒ x + y = 3y

⇒ x = 2y … (2)

The car covers x metres in six seconds. [Given]

That is, the car covers 2y metres in six seconds. [Using equation (2)]

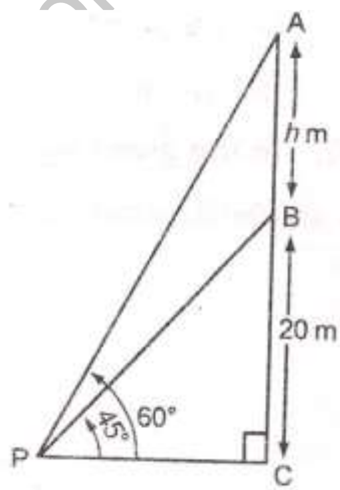
∴ The car covers y metres in three seconds.

Hence, the car will reach the foot of the tower in three seconds.

Q4. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find height of the tower.

Solution.

Let AB be the transmission tower fixed on the top of the building of height 20 m. Let AB = h m and .P be a point on ground, such that ∠BPC = 45°, ∠APC = 60°.



In rt. ΔPCB, ∠C = 90°

$$\therefore \quad \frac{BC}{PC} = \tan 45^\circ$$

$$\Rightarrow \quad \frac{20}{PC} = 1 \quad \Rightarrow \quad PC = 20m$$

In rt. ΔPCA, ∠C = 90°

$$\therefore \quad \frac{AC}{PC} = \tan 60^\circ$$

$$\Rightarrow \quad \frac{h+20}{20} = \sqrt{3} \Rightarrow h+20 = 20\sqrt{3}$$

$$\Rightarrow \quad h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

Hence, the height of the tower is 20 (√3 -1)m.