Triangles

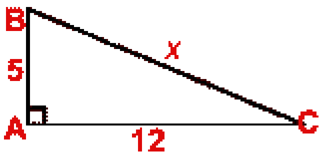
Level – 1 (1 MARK EACH)

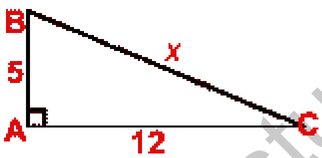
Fill in the blanks:

1. All ……………………triangles are similar. (Equilateral)
2. If a line divides two sides of a triangle in the same ratio, the line is ....to the third side. (Parallel)
3. Sides of two similar triangles are in the ratio 4:9.The ratio of areas of these triangles is ……………. (16:81)
4. Two triangle are said to be similar if................ (Corresponding sides are in same ratio and corresponding angles are equal)
5. Traingle have ....diagonals. (No)
6. The six elements of a triangle are its three angles and the ------------------ (three sides)

Level-2 (2 MARK EACH)

7. Find the length of x in the following cases:



Ans. 

In Δ ABC

$BC^2 = AB^2 + AC^2$ (by Pythagoras theorem)

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144 = 169$

$x = \sqrt{169}$

$x = 13$

8. A Ladder 25m long reaches a window of a building 20m above the ground. Determine the distance of the foot of the ladder from the building.

SOLUTION : suppose that AB is a ladder, B is the window and CB is the building. Then, triangle ABC is right angle at C,

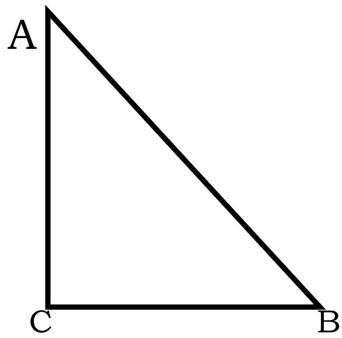
$AB^2 = AC^2 + BC^2$

$25^2 = AC^2 + 20^2$

$AC^2 = 625 – 400$

$= 225$

$AC = \sqrt{225} = 15m$



Hence, the foot of the ladder is at a distance 15 m from the building.

9. If Δ ABC ~ Δ DEF, such that AB = 1.2cm, and DE = 1.4cm. Find the ratio of areas of Δ ABC and Δ DEF.
Ans. Ratio of Areas of similar triangle is equal to ratio of square of corresponding sides.

Therefore $\frac{ar\Delta ABC}{ar\Delta DEF} = AB^2 / DE^2$

Or $\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{1.2}{1.4}\right)^2$

$= \frac{144}{196}$

$= \frac{36}{49}$

10. If the areas of two similar triangles are equal, prove that they are congruent.

Ans.Let us take two triangles ABC and PQR with equal areas.

Then, we have;

$$\frac{ar(ABC)}{ar(PQR)} = \frac{1}{1}$$

In this case;

$$\frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{1}{1}$$

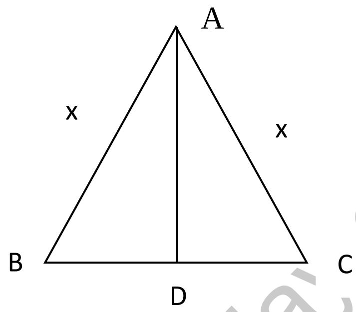
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{1}{1}$

AB = PQ and AC = PR
Angle A = angle P
Hence; the triangles are congruent.(BY SAS)

Level – 3 (3 MARK EACH)

11. In ΔABC, AB = AC = x, BC = 5 cm and the area of the triangle ABC is 15 cm². Find x.

Ans.



Construction AD ⊥ BC since ΔABC is an isosceles triangle therefore AD bisects BC
i:e BD = DC = 5/2
Area of ΔABC = ½ BC X AD

$= ½\ 5 \times AD = ½ \times 5 \times \sqrt{x^2 - BD^2}$

$15 = ½ \times 5 \times \sqrt{x^2 - BD^2}$

$\frac{15 \times 2}{5} = \sqrt{x^2 - \left(\frac{5}{2}\right)^2}$

Squaring both sides

$36 = x^2 - \frac{25}{4}$

$x^2 = 36 + \frac{25}{4}$

x = 6.5

<u>Alternative method</u>

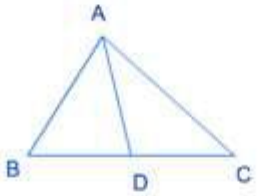
Area of ΔABC = ½ BC X AD
15 = ½ × 5 × AD
Or AD = 30/5= 6
In right ΔABD

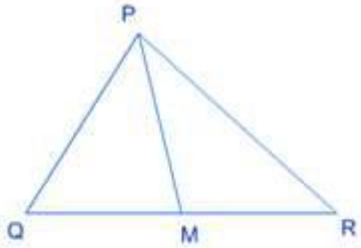
$x^2 = 36 + \frac{25}{4}$ (by Pythagoras theorem)

x= 6.5

12. If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $: \frac{AB}{PQ} = \frac{AD}{PM}$

Solution:





Δ ABC ~ Δ PQR (Given)

$\frac{AB}{PQ} = \frac{BC}{QR}$

$\frac{AB}{PQ} = \frac{2BD}{2QM}$

In Δ ABD & Δ PQM

$\frac{AB}{PQ} = \frac{BD}{QM}$

Angle B = Angle Q

Δ ABD ~ Δ PQM

Hence;

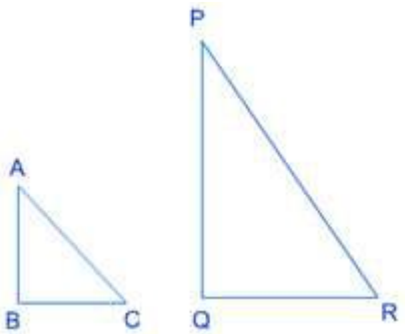
$$\frac{AB}{AD} = \frac{PQ}{PM}$$

(A side and the median of one triangle are in same ratio as a corresponding side and median of another triangle)

$: \frac{AB}{PQ} = \frac{AD}{PM}$

Proved

13. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.



Solution:

Height of pole = AB = 6 m and its shadow = BC = 4 m

Height of tower = PQ = ?and its shadow = QR = 28 m

The angle of elevation of the sun will be same at a given time for both the triangles.

Hence; ΔABC ~ ΔPQR

This means;

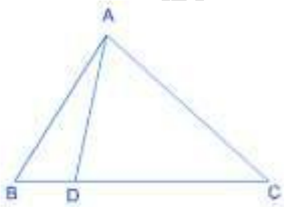
$$\frac{AB}{AC} = \frac{PQ}{QR}$$

$$Or, \frac{6}{4} = \frac{PQ}{28}$$

$$Or, PQ = \frac{6 \times 28}{4} = 42\,m$$

Height of tower = 42 m

14. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD.



Solution: In ΔBAC and ΔADC;

∠BAC = ∠ADC (given)

∠ACB = ∠DCA (Common angle)

Hence; ΔBAC ~ ΔADC

Hence;

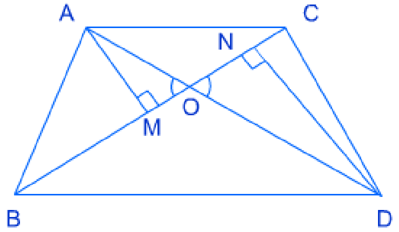
$$\frac{CA}{CB} = \frac{CD}{CA}$$
(corresponding sides are in same ratio)

Or, CA x CA = CB x CD

Or, CA^2 = CB x CD proved

Level – 4 (4 MARK EACH)

15. In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$



Solution: Let us draw altitudes AM and DN on BC; respectively from A and D

$$\frac{ar(ABC)}{ar(DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$$= \frac{AM}{DN}$$
In ΔAMO and ΔDNO;

∠ AMO = ∠ DNO (Right angle)

∠ AOM = ∠ DON (Opposite angles)

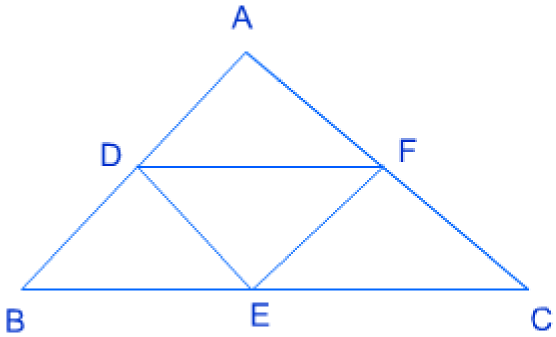
Therefore, ΔAMO ~ ΔDNO

Hence;

$$\frac{AM}{DN} = \frac{AO}{DO}$$

$$Or, \frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$$

16. D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.



Solution:

Since D, E and F are mid points of AB, BC and AC

Hence; ΔBAC ~ΔDFE

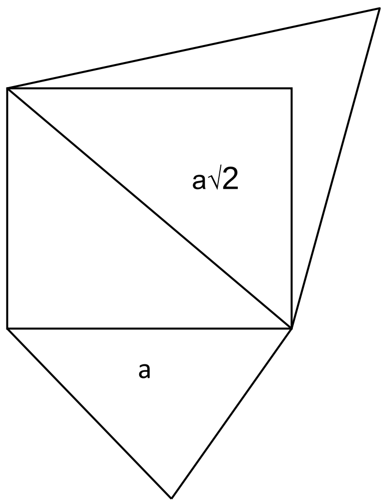
So,

$$\frac{DF}{BC}=\frac{EF}{AB}=\frac{DE}{AC}=\frac{1}{2}$$

So,

$$\frac{ar(DEF)}{ar(ABC)}=\frac{1^2}{2^2}=\frac{1}{4}$$

17. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.



Solution: **:** Let us take a square with side 'a'

Then the diagonal of square will be a$\sqrt{2}$

Area of equilateral triangle with side 'a'

$$=\frac{\sqrt{3}}{4}a^2$$

Area of equilateral triangle with side a√2

$$=\frac{\sqrt{3}}{4}\left(a\sqrt{2}\right)^2$$

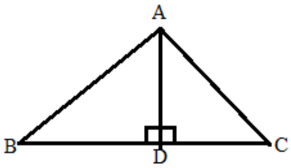
Ratio of two areas can be given as follows:

$$\frac{\frac{\sqrt{3}}{4} \times a^2}{\frac{\sqrt{3}}{4} \times 2a^2} = \frac{1}{2}$$

18. The perpendicular from A on side BC of ΔABC intersect BC at D such that DB=3CD. Prove that

    $2AB^2=2AC^2+BC^2$.

Ans:



Proof: AD $\perp$ BC
DB=3CD
BC=DB+CD=3CD+CD=4CD
CD=$\frac{1}{4}$ BC, DB=$\frac{3}{4}$ BC

In rt ΔABD; $AB^2=AD^2+BD^2$

$AB^2=AD^2+\left(\frac{3}{4}BC\right)^2$

$AB^2=AD^2+\frac{9}{16}BC^2$

In rt ΔACD, $AC^2=AD^2+CD^2$

$=AD^2+\left(\frac{1}{4}BC\right)^2$

$=AD^2+\frac{1}{16}BC^2$

$AB^2-AC^2=\frac{1}{2}BC^2$

$2AB^2-2AC^2=BC^2$
$2AB^2=2AC^2+BC^2$