UNIT 13

STATISTICS

Statistics are the only tools by which an opening can be cut through the formidable thicket of difficulties that bars the path of those who pursue the Science of Man.

1. Marks obtained by 70 students are given below:

Marks	20	70	50	60	75	90	40
No. of Students	8	12	18	6	9	5	12

Find the median. (Ans:50)

Ans:

Marks	No . of students	c.f
20	8	8
40	12	20
50	18	38
60	6	44
70	12	53
75	9	58
90	5	70

N = 70

$$\frac{N}{2} = \frac{70}{2} = 35$$

The corresponding value of marks for 35 is 50

2. The sum of deviations of a set of values $x_1, x_2, x_3, \ldots\ldots\ldots x_n$, measured from 50 is -10 and the sum of deviations of the values from 46 is 70.
   Find the value of n and the mean. (Ans:20,.49.5)

Ans: We have

$$\sum_{i=1}^{n}(X_i - 50) = -10 \text{ and } \sum_{i=1}^{n}(X_i - 46) = 70$$

$$\sum_{i=1}^{n} X_i - 50n = -10 \quad \ldots\ldots\ldots\ldots (1)$$

and $\sum_{i=1}^{n} X_i - 46_{\text{ m}} = 70 \ldots\ldots\ldots\ldots..(2)$

subtracting (2) from (1), we get

- 4 n = - 80 we get n = 20

$$\sum_{i=1}^{n} X_i - 50 \times 20 = -10$$

$$\sum_{i=1}^{n} X_i = 990$$

$$\text{Mean} = \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right) = \frac{990}{20} = 49.5$$

hence n = 20 and mean = 49.5

3. Prove that $\sum(x_i - \overline{x}) = 0$

Ans: To prove $\sum_{i=1}^{n}(X_i - \overline{X}) = 0$ algebraic sum of deviation from mean is zero

We have, $\overline{X} = \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)$

$$n\overline{X} = \sum_{i=1}^{n} X_i$$

Now, $\sum_{i=1}^{n}(X_i - \overline{X}) = (X_1 - \overline{X}) + (X_2 - \overline{X}) + \ldots\ldots\ldots + (X_n - \overline{X})$

$$\sum_{i=1}^{n}(X_i - \overline{X}) = (X_1 + X_2 + \ldots\ldots\ldots + X_n) - n\overline{X}$$

$$\sum_{i=1}^{n}(X_i - \overline{X}) = \sum_{i=1}^{n} X_i - n\overline{X}$$

$$\sum_{i=1}^{n}(X_i - \overline{X}) \quad = n\,\overline{X} \;-\; n\,\overline{X}$$

$$\sum_{i=1}^{n}(X_i - \overline{X}) = 0$$

Hence, $\sum_{i=1}^{n}(X_i - \overline{X}) = 0$

4.Compute the median from the following data

Mid value	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

(Ans:135.8)

Ans: Here , we are given the mid values. So, we should first find the upper and lower limits of the various classes. The difference between two consecutive values is h = 125 – 115 = 10

$\therefore$ Lower limit of a class = Midvalue - h /2

Upper limit = Midvalue + h / 2

Calculate of Median

Mid – value	Class Groups	Frequency	Cumulative frequency
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	160-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390
			$N = \Sigma f_i = 390$

We have,

N = 390 $\qquad \therefore$ N / 2 = 390 / 2 = 195

The cumulative frequency first greater than N i.e. 195 is 267 and the corresponding class is 150 – 160, so, 150 – 160 is the median class.

L = 150, f = 116, h = 10, f = 151

Now,

$$\text{Median} = L + \frac{\frac{n}{2} - f}{f} \times h$$

$$\text{Median} = 150 + \frac{195 - 151}{116} \times 10 = 153.8$$

5. The mean of 'n' observation is $\bar{x}$, if the first term is increased by 1, second by 2 and so on. What will be the new mean. (Ans: $\bar{x} + \frac{n+1}{2}$)

Ans: I term + 1
II term +2
III term + 3
.
.
n term + n

The Mean of the new numbers is $\bar{X} + \frac{\frac{n(n+1)}{2}}{n} = \bar{X} + \frac{(n+1)}{2}$

6. In a frequency distribution mode is 7.88, mean is 8.32 find the median. (Ans: 8.17)

Ans: Mode = 3 median – 2 mean
7.88 = 3 median – 2 x 8.32
7.88 +16.64 = 3 median

$\frac{24.52}{3}$ = median

$\therefore$ median = 8.17

7. The mode of a distribution is 55 & the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10.Find the frequency of the modal class. (Ans:15)

Ans: mode = 55
Modal class = 45 – 60
Modal class preceding $f_1 = 5$
After the modal class = $f_2 = 10$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

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$$55 = 45 + \frac{f-5}{2f-5-10} \text{ x } 15$$

$$10 = (\frac{f-5}{2f-15}) \text{ x } 15$$

$$\frac{10}{15} = \frac{f-5}{2f-15}$$

20 f - 150 = 15 f – 75

5 f = 75

$$f = \frac{75}{5} = 15$$

8. The mean of 30 numbers is 18, what will be the new mean, if each observation is increased by 2? (Ans:20)

Ans: Let x_1 , x_2, x_3 …… x_{30} be 30 number with then mean equal to 18 then

$$\bar{X} = \frac{1}{n}(\sum x_i)$$

$$18 = \frac{x_1 + x_2 + x_3 ...... + x_{30}}{30}$$

$x_1 + x_2 + x_{3+} …… + x_{30} = 18 \text{ x } 30 = 540$

New numbers are $x_1 + 2$, x_2, $+ 2$ $x_3 + 3$ …… $x_{30} + 2$

Let $\bar{X}$ be the mean of new numbers

then $$\bar{X} = \frac{(x_1+2)+(x_2+2)+......+(x_{30}+2)}{30}$$

$$\bar{X} = \frac{\frac{n(n+1)}{2}}{n}$$

$$\bar{X} = \frac{n+1}{2}$$

$$\frac{(x_1 + x_2 + ... + x_{30}) + 2X30}{30} = \frac{540+60}{30}$$

Mean of new numbers = $\frac{600}{30} = 20$

9. In the graphical representation of a frequency distribution if the distance between mode and mean is k times the distance between median and mean then find the value of k. (Ans:k=3)

Self Practice

10. Find the mean of 30 numbers given mean of ten of them is 12 and the mean of remaining 20 is 9. (Ans:10)

Ans: Total number of mean = 30

Mean of 10 is = 12

$$12 = \frac{\sum_{i=1}^{n} X_i}{10}$$

$\sum X_i = 12 \times 10 = 120$ ---(1)

Mean of 20 numbers is = 9

$$9 = \frac{\sum X_i}{20}$$

$$9 \times 20 = \sum_{i=1}^{n} X_i \quad \text{----- (2)}$$

$180 = = \sum X_i$

(1) + (2)

$$\text{Mean of 20 numbers} = \frac{120 + 180}{30}$$

$$= \frac{300}{30} = 10$$