1.Real numbers

(Key Points)

1. **Euclid's Division lemma:- G**iven Positive integers a and b there exist unique integers q and r satisfying
   a=bq +r, where 0≤r<b, where a, b, q and r are respectively called as dividend, divisor, quotient and remainder.

2. **Euclid's division Algorithm:-** To obtain the HCF of two positive integers say c and d, with c>0, follow the steps below:

   Step I: Apply Euclid's division lemma, to c and d, so we find whole numbers, q and r such that c =dq +r, 0≤ $r < d$.
   Step II: If r=0, d is the HCF of c and d. If r $\neq 0, apply\ the$ division lemma to d and r.
   Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF

3. **The Fundamental theorem of Arithmetic:-**
   Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.
   Ex.: $24 = 2\ X\ 2\ X\ 2\ X\ 3 = 3\ X\ 2\ X\ 2\ X\ 2$

   Theorem: LET x be a rational number whose decimal expansion terminates. Then x can be expressed in the form
   Of $\frac{p}{q}$ where $p\ and\ q$ are co-prime and the prime factorisation of q is the form of $2^n.5^m$, where n, m are non negative integers.
   Ex. $\frac{7}{10} = \frac{7}{2\ X\ 5} = 0.7$

   Theorem: LET x=$\frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form of $2^n.5^m$, where n, m are non negative integers. Then x has a decimal expansion which is non terminating repeating (recurring).
   Ex. $\frac{7}{6} = \frac{7}{2\ X\ 3} = 1.1666 \ldots \ldots ..$

   Theorem: For any two positive integers a and b,
   HCF (a,b) X LCM (a,b)=a X b
   Ex.: 4 & 6; HCF (4,6) = 2, LCM (4,6) = 12; HCF X LCM = 2 X 12 =24
   Ans. : a X b = 24

(Level- 1)

1. If $\frac{p}{q}$ is a rational number ($q \neq 0$). What is the condition on q so that the decimal representation of $\frac{p}{q}$ is terminating?

   Ans. q is form of $2^n.5^m$ $where\ n, m\ are\ non\ negative\ integers.$

2. Write a rational number between $\sqrt{2}\ and\ \sqrt{3}$.

   Ans. 1.5

3. The decimal expansion of the rational no. $\frac{43}{2^4.5^3}$ $will\ terminate\ after\ how\ many\ of\ decimals?$

   Ans. After 4 places of decimal.

4. Find the $(HCF\ X\ LCM)$ $for\ the\ numbers$ 100 and 190.

Ans. 19000

5. State whether the number $(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})$ *is* rational or irrational justify.

Ans. Rational

6. Write one rational and one irrational number lying between 0.25 and 0.32.

Ans. One rational no. =0.26, one irrational no. = 0.27010010001………

7. Express 107 in the form of 4q + 3 for some positive integer.

Ans. 4 X 26 + 3

8. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non terminating repeating decimal expansion.

Ans. Terminating.

(level - 2)

1. Use Euclid's division algorithm to find the HCF of 1288 and 575.

Ans. 23.

2. Check whether $5\ X\ 3\ X\ 11 + 11\ and\ 5\ X\ 7 + 7\ X\ 3 + 3$ are composite number and justify.

Ans. Composite number.

3. Check whether 6^n can end with the digit 0, where n is any natural number.

Ans. No, 6^n can not end with the digit 0.

4. Given that LCM (26, 169) = 338, write HCF (26, 169).]

Ans. 13

5. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.

Ans. HCF = 6
LCM = 360

(level - 3)

1. Show that $\sqrt{3}$ is an irrational number.
2. Show that $5 + 3\sqrt{2}$ is an irrational number.
3. Show that square of an odd positive integer is of the form 8m + 1, for some integer m.
4. Find the LCM & HCF of 26 and 91 and verify that $LCM\ X\ HCF = product\ of\ the\ two\ numbers.$

Ans. LCM=182, HCF=13

(PROBLEMS FOR SELF EVALUATION/HOTS)

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime factors.
3. Show that the square of an odd positive integers is of the form 8m + 1 for some whole number m.
4. Find the LCM and HCF of 17, 23 and 29.

5. Prove that $\sqrt{2}$ is not a rational number.
6. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5, 7 and 11 respectively.
7. Show that there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ *is rational*.
8. Using prime factorization method, find the HCF and LCM of 72, 126 and 168. Also show that $HCF\ X\ LCM\ \neq product\ of\ the\ three\ numbers$.

2. Polynomials

(Key Points)

Polynomial:

An expression of the form $p(x)$ =$a_0 + a_1x + a_2x^2 + \text{-----} + a_nx^n$ where $a_n \neq 0$ is called a polynomial in variable x of degree n. where; a_0, a_1, ----- a_n are real numbers and each power of x is a non negative integer.
Ex.:- $2x^2 - 5x + 1$ is a polynomial of degree 2.
Note: $\sqrt{x} + 3$ *is not a polynomial*.

- A polynomial $p(x) = ax + b$ *of degree* 1 *is called a linear polynomial*. Ex. 5x -3, 2x etc
- A polynomial $p(x) = ax^2 + bx + c$ *of degree* 2 *is called a quadratic polynomial*.Ex. $2x^2 + x - 1$, $1 - 5x + x^2$ etc.
- A polynomial $p(x) = ax^3 + bx^2 + cx + d$ *of degree* 3 *is called a cubic polynomial*.
  Ex. $\sqrt{3}x^3 - x + \sqrt{5},\ x^3 - 1$ etc.

Zeroes of a polynomial: A real number k is called a zero of polynomial $p(x)$ *if* $p(x) = 0$. The graph of $y = p(x)$ intersects the X- axis.

- A linear polynomial has only one zero.
- A Quadratic polynomial has two zeroes.
- A Cubic polynomial has three zeroes.

For a quadratic polynomial: If α, β are zeroes of $P(x) = ax^2 + bx + c$ then :

1. Sum of zeroes = α +β = $\frac{-b}{a} = \frac{-Coefficient\ of\ x}{coefficient\ of\ x^2}$
2. Product of zeroes = α . β = $\frac{c}{a} = \frac{Constant\ term}{coefficient\ of\ x^2}$

- A quadratic polynomial whose zeroes are α and β, is given by:
  $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$
  $= x^2 - (sum\ of\ zeroes)x + product\ of\ zeroes.$
- If α, β *and* γ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:
  - $\alpha + \beta + \gamma = \frac{-b}{a}$
  - $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
  - $\alpha\beta\gamma = \frac{-d}{a}$

Division algorithm for polynomials: If $p(x)$ *and* $g(x)$ *are any two polynomials with* $g(x) \neq 0$, then we can find polynomials $q(x)$ *and* $r(x)$ *such that*: