

QUADRATIC EQUATIONS

KEY POINTS

1. The general form of a quadratic equation is $ax^2+bx+c=0$, $a \neq 0$. a , b and c are real numbers.
2. A real number x is said to be a root of the quadratic equation $ax^2+bx+c=0$ where $a \neq 0$ if $ax^2+bx+c=0$. The zeroes of the quadratic equation polynomial $ax^2+bx+c=0$ and the roots of the corresponding quadratic equation $ax^2+bx+c=0$ are the same.
3. Discriminant:- The expression b^2-4ac is called discriminant of the equation $ax^2+bx+c=0$ and is usually denoted by D . Thus discriminant $D = b^2-4ac$.
4. Every quadratic equation has two roots which may be real, co incident or imaginary.
5. IF α and β are the roots of the equation $ax^2+bx+c=0$ then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{And } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

6. Sum of the roots, $\alpha + \beta = -\frac{b}{a}$ and product of the roots, $\alpha\beta = \frac{c}{a}$
7. Forming quadratic equation, when the roots α and β are given.
 $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
8. Nature of roots of $ax^2+bx+c=0$
 - i. If $D > 0$, then roots are real and unequal.
 - ii. $D = 0$, then the equation has equal and real roots.
 - iii. $D < 0$, then the equation has no real roots

LEVEL-I

1. IF $\frac{1}{2}$ is a root of the equation $x^2+kx-5/4=0$, then the value of K is
 (a) 2 [Ans(d)]
 (b) -2
 (c) $\frac{1}{4}$
 (d) $\frac{1}{2}$
2. IF $D > 0$, then roots of a quadratic equation $ax^2+bx+c=0$ are
 (a) $\frac{-b \pm \sqrt{D}}{2a}$ (b) $\frac{-b + \sqrt{D}}{2a}$ (c) $\frac{-b - \sqrt{D}}{2a}$ (d) None of these [Ans(a)]
3. Discriminant of $x^2+5x+5=0$ is
 (a) $5/2$ (b) -5 (c) 5 (d) -4 [Ans(c)]
4. The sum of roots of a quadratic equation $x^2+4x-320=0$ is
 (a) -4 (b) 4 (c) $1/4$ (d) $1/2$ [Ans(a)]
5. The product of roots of a quadratic equation $2x^2+7x-4=0$ is
 (a) $2/7$ (b) $-2/7$ (c) $-4/7$ (d) -2 [Ans(d)]
6. Values of K for which the equation $9x^2+2kx-1=0$ has real roots are:
 (a) $k \geq \pm 3$ (b) $k \geq 3$ or $k \leq -3$ (c) $k \geq -3$ (d) $k \leq \pm 3$ [Ans(b)]

LEVEL-II

1. For what value of k , $x=a$ is a solution of equation $x^2-(a+b)x+k=0$?

Ans. $K=ab$

2. Represent the situation in the form of quadratic equation:-

Rohan's mother is 26 years older than him. the product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Ans. $x^2+32x-273=0$ where x (in years) is Rohan's present age

3. Find the roots of $x^2-3x-10=0$

Ans. -2, 5

4. Find two consecutive positive integers, sum of whose squares is 365.

Ans. 13, 14

5. Find the roots of Quadratic equation $4x^2+4\sqrt{3}x+3=0$ by using the quadratic formula.

Ans. $-\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

6. Find the discriminant of the Quadratic equation $2x^2-4x+3=0$ and hence find the nature of its roots.

Ans. $D=-8<0$ its no real roots.

LEVEL - 3

1. If $x=2$ and $x=3$ are roots of the equation $3x^2-2kx+2m=0$ find the value of k and m .

Ans. $K=\frac{15}{2}, m=9$

2. Solve the equation:

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}, x \neq 0, x \neq -1$$

Ans. $x=\frac{3}{2}$ or $x=\frac{-5}{2}$

3. Solve the equation $2x^2-5x+3=0$ by the method of completing square.

Ans. $x=\frac{3}{2}$ or $x=1$

4. Using quadratic formula, solve the equation: $p^2x^2+(p^2-q^2)x-q^2=0$.

Ans. $x=-1$, or $x=\frac{q^2}{p^2}$

5. The sum of two numbers is 15, if the sum of their reciprocals is $\frac{3}{10}$, find the numbers.

Ans. 10 and 5

[LEVEL - 4]

1. In a class test, the sum of shefali's marks in maths and English is 30. Had she got 2 marks more in maths and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Marks in maths = 12, marks in English = 18 or, marks in maths = 13, marks in English = 17

2. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans. 15 hours, 25 hours.

3. Find the roots of equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{13}, x \neq -4, 7$

Ans. 1, 2

4. Solve the following equation for 'x' $9x^2-9(a+b)x+(2a^2+5ab+2b^2)=0$

$$\text{Ans. } \frac{2a+b}{3}, \frac{a+2b}{3}$$

5. If the roots of the equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b+c$.

Self Evaluation

- Find the value of p so that the equation $3x^2 - 5x + 2p = 0$ has equal roots. Also find the roots.
- The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers.
- Find a and b such that $x+1$ and $x+2$ are factors of the polynomials $x^3 + ax^2 - bx + 10$.
- Find the quadratic equation whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.
- A person on tour has Rs. 360 for his daily expenses. If he exceeds his tour program by four days, he must cut down his daily expenses by Rs 3 per day. Find the number of days of his tour program.
- Divide 29 into two parts so that the sum of squares of the parts is 425.
- Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$
- If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$