

CHAPTER 1

QUADRATIC EQUATIONS

KEY POINTS

1. The equation $ax^2 + bx + c = 0$, $a \neq 0$ is the standard form of a quadratic equation, where a , b and c are real numbers.
2. A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$. If $a\alpha^2 + b\alpha + c = 0$, the zeros of quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
3. If we can factorise $ax^2 + bx + c = 0$, $a \neq 0$ in to product of two linear factors, then the roots of the quadratic equation can be found by equating each factors to zero.
4. The roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided that $b^2 - 4ac \geq 0$.
5. A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, has ____
 - (a) Two distinct and real roots, if $b^2 - 4ac > 0$.
 - (b) Two equal and real roots, if $b^2 - 4ac = 0$.
 - (c) Two roots are not real, if $b^2 - 4ac < 0$.
6. A quadratic equation can also be solved by the method of completing the square.
 - (i) $a^2 + 2ab + b^2 = (a + b)^2$
 - (ii) $a^2 - 2ab + b^2 = (a - b)^2$
7. Discriminant of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by $D = b^2 - 4ac$.

MULTIPLE CHOICE QUESTIONS

- The general form of a quadratic equation is ($a \neq 0$)
 - $ax^2 + bx + c$
 - $ax^2 + bx + c = 0$
 - $ax + b$
 - $ax + b = 0$
- Number of solutions of a quadratic equation are :
 - 0
 - 1
 - 2
 - 3
- If the equation $x^2 - (2 + m)x + (-m^2 - 4m - 4) = 0$ has coincident roots, then
 - $m = 0, m = 1$
 - $m = 2, m = 2$
 - $m = -2, m = -2$
 - $m = 6, m = 1$
- Discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by
 - $\sqrt{b^2 - 4ac}$
 - $\sqrt{b^2 + 4ac}$
 - $b^2 - 4ac$
 - $b^2 + 4ac$
- Which is a quadratic equation?
 - $x + \frac{1}{x} = 2$
 - $x^2 + 1 = (x + 3)^2$
 - $x(x + 2)$
 - $x + \frac{1}{x}$
- If the roots of a quadratic equation are 2 and 3, then the equation is
 - $x^2 + 5x + 6 = 0$
 - $x^2 + 5x - 6 = 0$
 - $x^2 - 5x - 6 = 0$
 - $x^2 - 5x + 6 = 0$
- Roots of the equations $x^2 - 3x + 2 = 0$ are
 - 1, -2
 - 1, 2
 - 1, -2
 - 1, 2

8. If the roots of a quadratic equation are equal, then discriminant is
- (a) 1 (b) 0
(c) greater than 0 (d) less than zero.
9. If one root of $2x^2 + kx + 1 = 0$ is $-\frac{1}{2}$, then the value of 'k' is
- (a) 3 (b) -3
(c) 5 (d) -5
10. The sum of the roots of the quadratic $5x^2 - 6x + 1 = 0$ is
- (a) $\frac{6}{5}$ (b) $\frac{1}{5}$
(c) $-\frac{5}{6}$ (d) $-\frac{1}{5}$
11. The product of the roots of the quadratic equation $2x^2 + 5x - 7 = 0$ is
- (a) $\frac{5}{2}$ (b) $-\frac{7}{2}$
(c) $-\frac{5}{2}$ (d) $\frac{7}{2}$
12. If the roots of the quadratic $2x^2 + kx + 2 = 0$ are equal then the value of 'k' is
- (a) 4 (b) -4
(c) ± 4 (d) ± 16
13. If the roots of $4x^2 + 3px + 9 = 0$ are real and distinct then, the value of p is
- (a) $p \geq -4$ or $p \leq 4$ (b) $p < -4$ or $p > 4$
(c) $p \leq -4$ or $p \leq 4$ (d) $p \leq -4$ or $p \geq 4$

14. If the sum and product of roots of a quadratic equation are $-\frac{7}{2}$ and $\frac{5}{2}$ respectively, then the equation is
- (a) $2x^2 + 7x + 5 = 0$ (b) $2x^2 - 7x + 5 = 0$
 (c) $2x^2 - 7x - 5 = 0$ (d) $2x^2 + 7x - 5 = 0$
15. Which constant must be added or subtracted to solve the equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square
- (a) $\frac{1}{8}$ (b) $\frac{1}{64}$
 (c) $\frac{1}{16}$ (d) none

SHORT ANSWER TYPE QUESTIONS

16. If one root of the equation $x^2 + 7x + k = 0$ is -2 , then find the value of k and the other root.
17. For what value of ' k ' the equation $2x^2 + kx + 3 = 0$ has equal roots?
18. For what value of ' p ', the equation $3x^2 + px + 3 = 0$ has real roots?
19. The product of two consecutive odd integers is 63. Represent this in form of a quadratic equation.
20. Find the roots of the equation : $x + \frac{1}{x} = 4\frac{1}{4}$, $x \neq 0$.
21. Find the roots of the equation : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.
22. Divide 51 in to two parts such that their product is 378.
23. Find ' k ' so that $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ has equal roots. ($k \neq 12$).
24. If (-5) is a root of the equation $2x^2 + px - 15 = 0$ and the equation $p(x^2 + x) + k = 0$ has equal roots, find values of p and k .

25. Find the roots of the equation

$$\frac{1}{x+2} - \frac{1}{x} = 3, x \neq -2, x \neq 0.$$

26. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

27. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

28. Find the nature of the roots of the following quadratic equations. If roots are real, find them.

(a) $5x^2 - 3x + 2 = 0$.

(b) $2x^2 - 9x + 9 = 0$.

29. Sum of two numbers is 15, if sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

30. Solve the following quadratic equations

$$2\left(\frac{2x+3}{x-3}\right) - 25\left(\frac{x-3}{2x+3}\right) = 5 \quad x \neq 3, x \neq -\frac{3}{2}$$

31. $a^2x^2 + (a^2 - b^2)x - b^2 = 0$.

32. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$.

33. $abx^2 + (b^2 - ac)x - bc = 0$.

34. $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}, x \neq 2, x \neq 4$.

35. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, x \neq 7$.

36. $3x^2 + 2\sqrt{5}x - 5 = 0$.

37. $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}, a \neq 0, b \neq 0, x \neq 0, x \neq -(a+b)$.

38. A two digit number is such that the product of the digit is 35, when 18 is added to the number, the digits interchange their places. Find the number.
39. Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46, find the integers.
40. A motor boat whose speed is 9 km/h in still water goes 12 km down stream and comes back in a total time 3 hours. Find the speed of the stream.
41. A train travels 360 km at uniform speed. If the speed had been 5 km/hr more it would have taken 1 hour less for the same journey. Find the speed of the train.
42. The hypotenuse of right angled triangle is 6cm more than twice the shortest side. If the third side is 2 cm less than the hypotenuse, find the sides of the triangle.
43. By a reduction of Rs. 2 per kg in the price of sugar. Anita can purchase 2 kg sugar more for Rs. 224. Find the original price of sugar per kg.
44. Rs. 6500 were divided equally among a certain number of students. Had there been 15 more students, each would have got Rs. 30 less. Find the original number of students.
45. A fast train takes 3 hours less than a slow train in travelling 600 km. If the speed of fast train is 10 km/hr more than the speed of slow train, find the speed of both the trains.
46. A girl is twice as old as her sister. Four years hence, the product of their ages will be 160. Find their present ages.
47. Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages.
48. In a cricket match against Sri Lanka, Sehwag took one wicket less than twice the number of wickets taken by Unmukt. If the product of the number of wickets taken by these two is 15, find the number of wickets taken by each.
49. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days. Find the time taken by B to finish the work alone.

50. Two pipes running together can fill a cistern in $2\frac{8}{11}$ minutes. If one pipe takes 1 minute more than the other to fill the cistern, find the time in which each pipe would fill the cistern alone.

ANSWERS

- | | |
|---|--|
| 1. b | 2. c |
| 3. a | 4. c |
| 5. a | 6. d |
| 7. d | 8. b |
| 9. a | 10. a |
| 11. b | 12. c |
| 13. b | 14. a |
| 15. a | 16. $k = 10$, second root = -5 |
| 17. $\pm 2\sqrt{6}$ | 18. $p \geq 6$ or $p \leq -6$ |
| 19. $x^2 + 2x - 63 = 0$ | 20. $4, \frac{1}{4}$ |
| 21. $-\frac{5}{\sqrt{2}}, -\sqrt{2}$ | 22. $9, 42$ |
| 23. $k = 14$ | |
| 24. $7, \frac{7}{4}$ | 25. $\frac{-3 + \sqrt{3}}{3}, \frac{-3 - \sqrt{3}}{3}$ |
| 26. $10, 5$ | |
| 27. Hint : For equal roots $D = 0$. | 28. (a) Not real roots. |
| 29. $5, 10$ | (b) Roots are real, $3, \frac{3}{2}$. |

30. 6, 1
31. $-1, \frac{b^2}{a^2}$
32. $\frac{\sqrt{3}}{4}, -\frac{2}{\sqrt{3}}$
33. $\frac{c}{b}, -\frac{b}{a}$
34. $5, \frac{5}{2}$
35. 1, 2
36. $\frac{\sqrt{5}}{3}, -\sqrt{5}$
37. $-a, -b$
38. 57
39. 4, 5, 6
40. 3 km/hr.
41. 40 km/hr.
42. 26 cm, 24 cm, 10 cm
43. Rs. 16
44. 50
45. 40, 50.
46. 12, 6
47. 27 yrs., 5 yrs.
48. 30 days
50. 5, 6 min