

QUADRATIC EQUATIONS

IMPORTANT CONCEPTS:-

TAKE A LOOK:

1. The general form of a quadratic equation is $ax^2+bx+c=0$, $a \neq 0$. a , b and c are real numbers.
2. A real number x is said to be a root of the quadratic equation $ax^2+bx+c=0$ where $a \neq 0$ if $ax^2+bx+c=0$. The zeroes of the quadratic equation polynomial $ax^2+bx+c=0$ and the roots of the corresponding quadratic equation $ax^2+bx+c=0$ are the same.
3. Discriminant:- The expression b^2-4ac is called discriminant of the equation $ax^2+bx+c=0$ and is usually denoted by D . Thus discriminant $D = b^2-4ac$.
4. Every quadratic equation has two roots which may be real, co incident or imaginary.
5. IF α and β are the roots of the equation $ax^2+bx+c=0$ then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{And } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

6. Sum of the roots, $\alpha + \beta = -\frac{b}{a}$ and product of the roots, $\alpha \cdot \beta = \frac{c}{a}$
7. Forming quadratic equation, when the roots α and β are given.

$$x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$$
8. Nature of roots of $ax^2+bx+c=0$
 - i. If $D > 0$, then roots are real and unequal.
 - ii. $D = 0$, then the equation has equal and real roots.
 - iii. $D < 0$, then the equation has no real roots

LEVEL-I

1. IF $\frac{1}{2}$ is a root of the equation $x^2+kx-5/4=0$, then the value of K is
 (a) 2 [Ans(d)]
 (b) -2
 (c) $\frac{1}{4}$
 (d) $\frac{1}{2}$
2. IF $D > 0$, then roots of a quadratic equation $ax^2+bx+c=0$ are
 (a) $\frac{-b \pm \sqrt{D}}{2a}$ [Ans(a)]
 (b) $\frac{-b + \sqrt{D}}{2a}$
 (c) $\frac{-b - \sqrt{D}}{2a}$
 (d) None of these
3. Discriminant of $x^2+5x+5=0$ is
 (a) $5/2$ (b) -5 (c) 5 (d) -4 [Ans(c)]
4. The sum of roots of a quadratic equation $x^2+4x-320=0$ is
 (a) -4 (b) 4 (c) $1/4$ (d) $1/2$ [Ans(a)]
5. The product of roots of a quadratic equation $2x^2+7x-4=0$ is
 (a) $2/7$ (b) $-2/7$ (c) $-4/7$ (d) -2 [Ans(d)]
6. Values of K for which the equation $9x^2+2kx-1=0$ has real roots are:
 (a) $k \geq \pm 3$ (b) $k \geq 3$ or $k \leq -3$ (c) $k \geq -3$ (d) $k \leq \pm 3$ [Ans(b)]

LEVEL-II

1. Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$

$$[\text{Ans- } x = \frac{2\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}]$$

2. The sum of the squares of two consecutive odd number is 394. Find the numbers.

$$[\text{Ans- } 13, 15 \text{ or } -15, 13]$$

3. Find the root of $6x^2 + \sqrt{2}x - 2 = 0$ by factorization.

$$[\text{Ans- } x = \frac{-\sqrt{2}}{3}, \frac{\sqrt{2}}{2}]$$

4. The sum of two numbers is 8 . Determine the numbers if the sum of their reciprocals is $8/15$.

$$[\text{Ans- } 5 \text{ and } 3 \text{ or } 3 \text{ and } 5]$$

5. For what value of k does $(k-12)x^2 + 2(k-12)x + 2 = 0$ has equal roots?

$$[\text{Ans- } k=14]$$

6. Divide 51 into two parts whose product is 378.

$$[\text{Ans- } 9, 42]$$

LEVEL-III

1. For what value of k, will the equation $2x^2 - 2(1+2k)x + (3+2k) = 0$ have real but distinct roots ? When will the roots be equal?

$$[\text{Ans } k < -5/2 \text{ or } k > \sqrt{5}/2, k = \pm \sqrt{5}/2]$$

2. Solve for x: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$. [Ans $\sqrt{3}/4, -2/\sqrt{3}$]

3. Using quadratic formula solve the following quadratic equation for x: $x^2 - 2ax + (a^2 - b^2) = 0$

$$[\text{Ans } a+b, a-b]$$

4. The speed of a boat in still water is 11 km/hr . It can go 12 km up stream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.

$$[5 \text{ km/h}]$$

5. Solve for x: $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$$[\text{Ans } 1/b^2, -1/a^2]$$

6. Solve the following quadratic equation for x:

$$x^2 - 2(a+2)x + (a+1)(a+3) = 0$$

$$[\text{Ans- } a+1, a+3]$$

LEVEL-IV

1. Solve for $\frac{1}{a+b-x} = \frac{1}{b} + \frac{1}{b} + \frac{1}{x}$; $a \neq 0, b \neq 0, x \neq 0$

[Ans. -a, -b]

2. An aeroplane left 30 minutes later than its scheduled time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Determine its usual speed.

[Ans 750 km/hr]

3. Using the quadratic formula solve the equation $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

[Ans $\frac{3a^2}{b^2}, \frac{4b^2}{a^2}$]

4. Solve for x: $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$ ($x \neq 2, 4$)

[Ans 5/2, 5]

5. If (-5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots then find the values of P and K.

[Ans -24m, 8m]

6. The sum of the areas of two squares is $640m^2$. If the difference of their perimeters is 64m. Find the sides of the two squares.

[Ans 24m, 8m]

SELF EVALUATION

1. If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then prove that $2b = a + c$

2. Solve by using quadratic formula

$$(x^2 + 3x + 2)^2 - 8(x^2 + 3x) - 4 = 0$$

3. If α and β are the roots of the equation $lx^2 - mx + n = 0$, Find the equation whose roots are α / β and $b\beta / \alpha$.

4. The difference of two numbers is 5 and the difference of their reciprocal is $1/10$. Find the numbers.