## TOPIC QUADRATIC EQUATIONS:

## LEVEL 1 (1 Mark each)

1. What will be the nature of roots of the quadratic equation: $2 x^{2}+4 x-7=0$ ?

Sol. $D=b^{2}-4 a c$

$$
\begin{aligned}
& =4^{2}-4 \times 2 \times(-7) \\
& =16+56=76>0
\end{aligned}
$$

Hence, roots of quadratic equation are real and unequal.
2. Show that $x=-2$ is a solution of $3 x^{2}+13 x+14=0$

Sol. Put the value of x in the quadratic equation
L.H.S $=3 x^{2}+13 x+14$

$$
=3(-2)^{2}+13(-2)+14
$$

$12-26+14=0=$ R.H.S
Hence, $x=-2$ is a solution of $3 x^{2}+13 x+14=0$
3. Check if $(x+1)^{2}=2(x-3)$ is a quadratic or not?

Sol. $(x+1)^{2}=2(x-3)$

$$
\begin{gathered}
x^{2}+2 x+1=2 x-6 \\
x^{2}+7=0
\end{gathered}
$$

Which is a quadratic.
4. Find the roots of the quadratic $2 x^{2}-7 x+3=0$

$$
\begin{aligned}
& \text { Sol. } 2 x^{2}-7 x+3=0 \\
& 2 x^{2}-6 x-x+3=0 \\
& 2 x(x-3)-(x-3)=0 \\
& (2 x-1)(x-3)=0
\end{aligned}
$$

$$
\begin{gathered}
(2 x-1)=0 \text { or } x-3=0 \\
x=\frac{1}{2}, x=3
\end{gathered}
$$

LEVEL 2 (2 Marks each)

1. Find the value of p for which the quadratic equation $4 x^{2}+p x+3=0$ has equal roots.

Sol. For equal roots

$$
D=0
$$

$$
b^{2}-4 a c=0
$$

i.e., $p^{2}-4 \times 4 \times 3=0$

$$
\begin{gathered}
p^{2}-48=0 \\
p^{2}=48 \\
p= \pm 4 \sqrt{3}
\end{gathered}
$$

2. Find two consecutive positive integers, sum of whose squares is 925 .

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Sol. Let two consecutive positive integers are x and $\mathrm{x}+1$
A.T.Q

$$
\begin{gathered}
x^{2}+(x+1)^{2}=925 \\
x^{2}+x^{2}+2 x+1=925 \\
2 x^{2}+2 x+1=925 \\
2 x^{2}+2 x-924=0 \\
x^{2}+x-462=0 \\
x^{2}+12 x-11 x-462=0 \\
x(x+12)-11(x+12)=0 \\
(x+12)(x-11)=0 \\
x=-12 \text { or } x=11
\end{gathered}
$$

$x=-12$ is not possible
Therefore integers are 11 and $11+1$ or 11 and 12 .
3. Solve the quadratic $2 x^{2}-7 x+3=0$ by using quadratic formula.

Sol. We have $2 x^{2}-7 x+3=0$
Here $a=2, b=-7$ and $c=3$
Therefore $D=b^{2}-4 a c$

$$
\begin{gathered}
D=(-7)^{2}-4 \times 2 \times 3 \\
=49-24=25>0
\end{gathered}
$$

Therefore roots exist.
Now $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad x=\frac{-(-7) \pm \sqrt{25}}{2 \times 2}$

$$
\begin{gathered}
x=\frac{7 \pm 5}{4} \\
x=\frac{12}{4} \text { or } x=\frac{2}{4} \\
x=3 \text { or } x=\frac{1}{2}
\end{gathered}
$$

4. For what value of $k$, is 3 a root of the equation $2 x^{2}+x+k=0$ ?

Sol. 3 is root of $2 x^{2}+x+k=0$ then,
$2(3)^{2}+3+k=0$
$18+3+k=0$
$21+k=0$.
$\mathrm{K}=-21$
LEVEL 3 (3-Marks each)

1. Solve $4 x^{2}+4 \sqrt{3} x+3=0$ by the method of completing squares.

Sol. Given equation is $4 x^{2}+4 \sqrt{3} x+3=0$
Dividing both sides by 4

$$
\begin{gathered}
x^{2}+\sqrt{3} x+\frac{3}{4}=0 \\
x^{2}+\sqrt{3} x=-\frac{3}{4}
\end{gathered}
$$

Adding square of half the coefficient of x to both sides

$$
\begin{gathered}
x^{2}+\sqrt{3} x+\left(\frac{\sqrt{3}}{2}\right)^{2}=-\frac{3}{4}+\left(\frac{\sqrt{3}}{2}\right)^{2} \\
\left(x+\frac{\sqrt{3}}{2}\right)^{2}=0 \\
\left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0 \\
x=-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}
\end{gathered}
$$

Hence the roots of the equation $4 x^{2}+4 \sqrt{3} x+3=0$ are $-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$
2. Had Anita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?
Sol. Let her actual marks be $x$
Therefore, $9(x+10)=x^{2}$

$$
\begin{aligned}
& \Rightarrow x^{2}-9 x-90=0 \\
& \Rightarrow x^{2}-15 x+6 x-90=0 \\
& \Rightarrow x(x-15)+6(x-15)=0 \\
& \Rightarrow(x+6)(x-15)=0
\end{aligned}
$$

Therefore, $x=-6$ or $x=15$
Since $x$ is the marks obtained, $x \neq-6$. Therefore, $x=15$.
So, Ajita got 15 marks in her mathematics test.
3. Solve for x ; $\frac{1}{2 x-3}+\frac{1}{x-5}=1, x \neq \frac{3}{2}, 5$

Sol.

$$
\begin{gathered}
\frac{1}{2 x-3}+\frac{1}{x-5}=1 \\
\frac{(x-5)+(2 x-3)}{(2 x-3)(x-5)}=1 \\
\frac{3 x-8}{2 x^{2}-13 x+15}=1 \\
2 x^{2}-16 x+23=0 \\
a=2, b=-16, c=23 \\
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
=\frac{16 \pm \sqrt{(-16)^{2}-4 \times 2 \times 23}}{2 \times 2}
\end{gathered}
$$

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$$
\begin{aligned}
& =\frac{16 \pm \sqrt{72}}{4} \\
& x=\frac{8 \pm 3 \sqrt{2}}{2}
\end{aligned}
$$

4. A cottage industry produces a certain number of articles in a day. It was observed on a particular day that the cost of production of each article war 3 more than the twice the number of articles produced on that day. If the total cost of production on that day was Rs 90 , find the number of articles produced and the cost of each article.

Solution: Let the number of articles be x
Therefore the cost of production $=3+2 \mathrm{x}$

$$
\text { Total cost of production = Rs. } 90
$$

According to questions:

$$
\begin{gathered}
x(3+2 x)=90 \\
2 x^{2}+3 x-90=0 \\
2 x^{2}+15 x-12 x-90=0 \\
x(2 x+15)-6(2 x+15)=0 \\
(2 x+15)(x-6)=0 \\
x=-\frac{15}{2} \text { or } x=6
\end{gathered}
$$

$x=-\frac{15}{2}$ is not possible so $x=6$
Hence number of articles produced $=6$
And cost of production of each article is 15.

## LEVEL 4 (4 -Marks each)

1. A motor boat whose speed is $18 \mathrm{~km} / \mathrm{h}$ in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let the speed of the stream be $x \mathrm{~km} / \mathrm{h}$.
Therefore, the speed of the boat upstream $=(18-x) \mathrm{km} / \mathrm{h}$ and the speed of the boat downstream $=(18+x) \mathrm{km} / \mathrm{h}$.
The time taken to go upstream $=\frac{\text { distance }}{\text { speed }}=\frac{24}{18-x}$ hours.
Similarly, the time taken to go downstream $=\frac{24}{18-x}$ hours.
According to the question,

$$
\begin{aligned}
\frac{24}{18-x}-\frac{24}{18+x} & =1 \\
& \Rightarrow 24(18+x)-24(18-x)=(18-x)(18+x) \\
& \Rightarrow 324+24 \mathrm{x}-324+24 \mathrm{x}=324-\mathrm{x}^{2} \\
& \Rightarrow x^{2}+48 x-324=0
\end{aligned}
$$

Using the quadratic formula, we get

$$
\begin{gathered}
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
x=\frac{-48 \pm \sqrt{48^{2}-4 \times 1 \times(-324)}}{2 \times 1} \\
x=\frac{-48 \pm \sqrt{3600}}{2} \\
x=\frac{-48 \pm 60}{2} \\
x=\frac{-48+60}{2}, x=\frac{-48-60}{2} \\
x=6,-54
\end{gathered}
$$

Since x is the speed of stream it cannot be negative. So, we ignore the root $x=-54$.
Therefore, $x=6$ gives the speed of the stream as $6 \mathrm{~km} / \mathrm{h}$.
2. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of $6 \mathrm{~km} / \mathrm{h}$ more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?
Sol. Let its original average speed be $x \mathrm{~km} / \mathrm{h}$. Therefore,
A.T.Q

$$
\begin{gathered}
\frac{63}{x}+\frac{72}{x+6}=3 \\
63(x+6)+72 x=3 x(x+6) \\
63 x+378+72 x=3 x^{2}+18 x \\
3 x^{2}-117 x-378=0 \\
3 x^{2}-39 x-126=0 \\
x^{2}-42 x+3 x-126=0 \\
(x+3)(x-42)=0 \\
x=-3 \text { or } x=42
\end{gathered}
$$

Since $x$ is the average speed of the train, $x$ cannot be negative. Therefore, $x=42$
So, the original average speed of the train is $42 \mathrm{~km} / \mathrm{h}$.
3. A two digit number is such that the product of its digits is 18 . When 63 is subtracted from the number the digits interchange their places. Find the number.

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Sol.let the digit at tens place be x , then digit at unit place $=\frac{18}{x}$.
Therefore number $=10 x+\frac{18}{x}$
and the number obtained by interchanging the digit $=10 \times \frac{18}{x}+x$
A.T.Q
$\left(10 x+\frac{18}{x}\right)-63=10 \times \frac{18}{x}+x$
$\left(10 x+\frac{18}{x}\right)-\left(10 \times \frac{18}{x}+x\right)=63$

$$
\begin{gathered}
10 x+\frac{18}{x}-\frac{180}{x}-x=63 \\
x^{2}-7 x-18=0 \\
x^{2}-9 x+2 x-18=0 \\
(x-9)(x+2)=0 \\
x=9,-2
\end{gathered}
$$

Therefore $x=9 \quad$ (as a digit cannot be negative)
Hence the required number $=10 \times 9+\frac{18}{9}=92$
4. A train takes 2 hours less for the journey of 300 km , if its speed is increased by $5 \mathrm{~km} / \mathrm{h}$ from its usual speed. Find the usual speed of train.
Sol. Let the usual speed of train $=x \mathrm{~km} / \mathrm{h}$
Therefore, time taken to cover $300 \mathrm{~km}=\frac{300}{x}$ hours
When its speed is increased by $5 \mathrm{~km} / \mathrm{h}$, then time taken by the train tom cover the distance of $300 \mathrm{~km}=\frac{300}{x+5}$ hours
According to the question,

$$
\begin{gathered}
\frac{300}{x}-\frac{300}{x+5}=2 \\
\frac{300(x+5)-300 x}{x(x+5)}=2 \\
\frac{300 x+1500-300 x}{x^{2}+5 x}=2 \\
x^{2}+5 x-750=0 \\
(x-25)(x+30)=0 \\
x=25 \text { or } x=-30
\end{gathered}
$$

$x=25$ (as speed cannot be negative)
Therefore speed of the train $=25 \mathrm{~km} / \mathrm{h}$

