Chapter - 2

(Polynomials)

Key Concepts

Constants : A symbol having a fixed numerical value is called a constant.

Example : 7, 3, -2, 3/7, etc. are all constants.

Variables : A symbol which may be assigned different numerical values is known as variable.

Example : $C = 2\pi\, r$ C - circumference of circle

r - radius of circle

Where 2 & π are constants. while C and r are variable

Algebraic expressions : A combination of constants and variables. Connected by some or all of the operations +, -, X and ÷ is known as algebraic expression.

Example : $4 + 9x - 5x^2y + \frac{3}{8}xy$ etc.

Terms : The several parts of an algebraic expression separated by '+' or '-' operations are called the terms of the expression.

Example : $x^3 + 2x^2y + 4xy^2 + y^3 + 7$ is an algebraic expression containing 5 terms $x^3, 2x^3y,\ -4xy^2,\ y^3$ & 7

Polynomials : An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

(i) $5x^3 - 4x^2 - 6x - 3$ is a polynomial in variable x.

(ii) $5 +\ 8x^{3/2} +\ 4x^{-2}$ is an expression but not a polynomial.

Polynomials are denoted by $p(x),\ \ q(x) and\ r(x) etc.$

Coefficients : In the polynomial $x^3 + 3x^2 + 3x + 1$, coefficient of $x^3, x^2, x\ are\ 1, 3, 3$ respectively and we also say that +1 is the constant term in it.

Degree of a polynomial in one variable : In case of a polynomial in one variable the highest power of the variable is called the degree of the polynomial.

Classification of polynomials on the basis of degree.

	degree	Polynomial	Example
(a)	1	Linear	$x+1,\ 2x+3\ etc.$
(b)	2	Quadratic	ax^2+bx+c etc.
(c)	3	Cubic	x^3-3x^2+1 etc.
(d)	4	Biquadratic	x^4-1

Classification of polynomials on the basis of no. of terms

	No. of terms	Polynomial & Examples.
(i)	1	Monomial - $5, 3x, \frac{1}{3}y$ etc.
(ii)	2	Binomial - $(3+6x),\ (x-5y)$ etc.
(iii)	3	Trinomial- $2x^2+4x+2$ etc.

Constant polynomial : A polynomial containing one term only, consisting a constant term is called a constant polynomial the degree of non-zero constant polynomial is zero.

Zero polynomial : A polynomial consisting of one term, namely zero only is called a zero polynomial.

The degree of zero polynomial is not defined.

Zeroes of a polynomial : Let $p(x)$ be a polynomial. If $p(\alpha)=0,$ then we say that α is a zero of the polynomial of p(x).

Remark : Finding the zeroes of polynomial p(x) means solving the equation p(x)=0.

Remainder theorem : Let $f(x)$ be a polynomial of degree $n \geq 1$ and let a be any real number. When $f(x)$ is divided by $(x-a)$ then the remainder is $f(a)$

Factor theorem : Let $f(x)$ be a polynomial of degree $n>1$ and let a be any real number.

(i) If $f(a)=\ 0\ then\ (x-a) is\ factor\ of\ f(x)$

(ii) $If\ (x-a) is\ a\ factor\ of\ f(x)\ \ then\ f(a)=0$

Factor : A polynomial $p(x)$ is called factor of $q(x),\ if\ p(x)$ divides $q(x)$ exactly.

Factorization : To express a given polynomial as the product of polynomials each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

Example : $x^2 - 16 = (x+4)(x-4)$

Methods of Factorization :

Factorization by taking out the common factor

e.g.

$$36q^3b - 60a^2bc = 12a^2b\,(3a - 5c)$$

Factorizing by grouping

$$\begin{aligned} ab + bc + ax + cx &= (ab + bc) + (ax + cx) \\ &= b\,(a + c) + x\,(a + c) \\ &= (a + c)\,(b + x) \end{aligned}$$

Factorization of quadratic trinomials by middle term splitting method.

$$\begin{aligned} x^2 + bc + c &= x^2 + (p + q)x + pq \\ &= (x + p)(x + q) \end{aligned}$$

Identity : Identity is a equation (trigonometric, algebraic) which is true for every value of variable.

Some algebraic identities useful in factorization:

(i) $(x + y)^2 = x^2 + 2xy + y^2$

(ii) $(x - y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x - y)(x + y)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$

(vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 = 3xyz \quad if \;\; x + y + z = 0$

Section - A

Q.1 Which of the following expressions is polynomial?

(i) $x^5 - 2x^3 + x + 7$ (ii) $y^3 - \sqrt{3}y$

(iii) $5\sqrt{z} - 6$ (iv) $x - \frac{1}{x}$ (v) $x^{108} - 1$

(vi) $\sqrt[3]{x} - 27$ (vii) $x^{-2} + 2x^{-1} + 3$

Q.2 Write the degree of each of the following polynomial.

(i) $2x - \sqrt{5}$ (ii) $3 - x + x^2 - 6x^3$ (iii) 9

(iv) $8x^4 - 36x + 5x^7$ (v) $x^9 - x^5 + 3x^{10} + 8$ (vi) $2 - 3x^2$

Q.3 (i) Give an example of a binomial of degree 27.

(ii) Give an example of a monomial of degree 16.

(iii) Give an example of trinomial of degree 3.

Section - B

Q.4 If $p(x) = 5 - 4x + 2x^2$ $find\ (i)\ p(0)\ (ii)\ p(3)\ (iii)\ p(-2)$

Q.5 Find the zeros of the polynomials given below :

(i) $p(x) = x - 5$ (ii) $q(x) = x + 4$ (iii) $h(x) = 6x - 1$

(iv) $p(x) = ax + b$ (v) $r(x) = x^2 + 3x$ (vi) $l(x) = x^2 + 2x + 1$

Q.6 Find the remainder when $f(x) = 12x^3 - 13x^2 - 5x + 7$ is divided by $(3x + 2)$?

Q.7 Show that $(x + 5)$ is a factor of the polynomial

$f(x) = x^3 + x^2 + 3x + 115$

Q.8 Find the value of a for which $(x - a)$ is a factor of the polynomial.

$f(x) = x^5 - a^2x^3 + 2x + a - 3$

Section - C

Q.9 Factorize the following expressions.

(i) $5x^2 - 20xy$

(ii) $5a\,(b + c) - 7b\,(b + c)$

(iii) $x(x - y)^2 + 3x^2y\,(x - y)$

(iv) $6ab - b^2 + 12ac - 2bc$

(v) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

Q.10 Factorize :

(i) $9x^2 - 16y^2$ (ii) $x^3 - x$

Q.11 Factorize:

$a(a - 1) - b\,(b - 1)$

Q.12 Factorize following expressions.

(i) $x^2 + 9x + 18$

(ii) $x^2 - 4x - 21$

(iii) $x^2 - 9x + 18$

(iv) $x^2 - 19x + 78$

Q.13 Calculate $(997)^2$ using algebraic identities.

Q.14 Calculate 103 X 107 using algebraic identities.

Q.15 Expand $(2a + 3b + 4c)^2$.

Q.16 Factorize $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$.

Q.17 Expand (i) $(4a + 5b)^3$ (ii) $(5x - 3y)^3$

Q.18 Evaluate (i) $(95)^3$ (ii) $(106)^3$

Q.19 Factorize (i) $x^3 + 64$ (ii) $27x^3 + 125y^3$

(iii) $8a^3 - 27b^3$ (iv) $1 - 64a^3$

Section - D

Q.20 Factorize

$a^3 + 27b^3 + 8c^3 - 18abc$

Q.21 Factorize

$(p-q)^3 + (q-r)^3 + (r-p)^3$

Q.22 Find the product

$(3x - 5y - 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$

Q.23 If $If\ x + y + z = 9\ and\ xy + yz + zx = 23$ then find the value of

$(x^3 + y^3 + z^3 - 3xyz)$?

Self Evaluation

Q.24 Which of the following expression is a polynomial?

(a) $\sqrt{x} - 1$ (b) $\frac{x-1}{x+1}$ (c) $x^2 - \frac{2}{x^2} + 5$ (d) $x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6$

Q.25 Degree of zero polynomial is

(a) 1 (b) 0 (c) not defined (d) none of these

Q.26 For what value of k is the polynomial $p(x) = 2x^3 - kx^2 + 3x + 10$ exactly divisible by $(x + 2)$?

(a) $\frac{-1}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) -3

Q.27 The zeroes of the polynomial $p(x) = 3x^2 - 1\ are$

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}\ and\ \frac{-1}{\sqrt{3}}$

Q.28 If $\frac{x}{y} + \frac{y}{x} = -1$ where $x \neq 0,\ y \neq 0\ \ then\ find\ the\ value\ of\ x^3 - y^3$.

Q.29 If $(x + 2) and\ (x - 1) are\ factors\ of\ (x^3 + 10x^2 + m\,x + n)$ then find value of m & n?

Q.30 Find the value of $(369)^2 - (368)^2$

Q.31 Find value of 104 X 96

Q.32 If $a + b + c = 0$ find value of $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right)$?

Answers

Q.1 (i), (ii), (v)

Q.2 (i) 1 (ii) 3 (iii) 0 (iv) 4 (v) 9 (vi) 2

Q.4 (i) p (0) = 5 (ii) p(3) = 11 (iv) 21

Q.5 (i) $x = 5$ (ii) $x = -4$ (iii) x= 1/6 (iv) $x = -b/a$

(v) $x = 0, x = -3$ (vi) $x = -1, -1$

Q.6 remainder = 1

Q.8 a = 1

Q.9 (i) $5x(x - 4y)$

(ii) $(b + c)(5a - 7b)$

(iii) $x(x - y)[(x - y) + 3xy]$

(iv) $(b + 2c)(6a - b)$

(v) $\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

Q.10 (i) $(3x + 4y)\,(3x - 4y)$ (ii) $x(x + 1)(x - 1)$

Q.11 (a-b) (a+b-1)

Q.12 (i) $(x + 6)(x + 3)$

(ii) $(x - 7)(x + 3)$

(iii) $(x - 6)(x - 3)$

(iv) $(x - 6)(x - 13)$

Q.13 994009

Q.14 11021

Q.15 $4a^2 + 9b^2 + 16c^2 + 12ab + 24bc + 16ac$

Q.16 $(2x - y + z)^2$

Q.17 (i) $64a^3 + 125b^3 + 240a^2b + 300ab^2$

(ii) $125x^3 - 27y^3 - 225x^2y + 135xy^2$

Q.18 (i) 857375 (ii) 1191016

Q.19 (i) $(x + 4)(x^2 - 4x + 16)$

(ii) $(3x+5y)(9x^2-15xy+25y^2$

(iii) $(2a - 3b)(4a^2 + 6ab + 9b^2)$

(iv) $(1-4a)(1+4a+16a^2)$

Q.20 $(a+3b+2c)(a^2+9b^2+4c^2-3ab-6bc-2ac)$

Q.21 $3(p-q)(q-r)(r-p)$

Q.22 $27x^3-125y^3-64-180xy.$

Q.23 108

Q.24 $(d)\ \ x^2+\frac{2x^{\frac{3}{2}}}{x^{\frac{1}{2}}}+6$

Q.25 (c) not defined

Q.26 (d) -3

Q.27 (d) $\frac{1}{\sqrt{3}}$ & $\frac{-1}{\sqrt{3}}$

Q.28 0

Q.29 m = 7, n = -18

Q.30 737

Q.31 9984

Q.32 3
