CHAPTER 2

POLYNOMIALS

KEY POINTS

1. Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
2. A quadratic polynomial in x with real coefficient is of the form $ax^2 + bx + c$, where a, b, c are real number with $a \neq 0$.
3. The zeroes of a polynomial $p(x)$ are precisely the x–coordinates of the points where the graph of $y = p(x)$ intersects the x-axis *i.e.* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$.
4. A polynomial can have at most the same number of zeros as the degree of polynomial.
5. For quadratic polynomial $ax^2 + bx + c$ (a $\neq$ 0)

   Sum of zeros $= -\frac{b}{a}$

   Product of zeros $= \frac{c}{a}$.
6. The division algorithm states that given any polynomial $p(x)$ and polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that :

   $$p(x) = g(x).q\ (x) + r(x),\ g(x) \neq 0$$

   where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.

MULTIPLE CHOICE QUESTIONS

1. *A* real no. α is a zero of the polynomial $f(x)$ if

   (a) $f(\alpha) > 0$ (b) $f(\alpha) = 0$

   (c) $f(\alpha) < 0$ (d) none

2. The zeros of a polynomial $f(x)$ are the coordinates of the points where the graph of $y = f(x)$ intersects

   (a) x-axis (b) y-axis

   (c) origin (d) (x, y)

3. If β is 0 zero of $f(x)$ then ____ is one of the factors of $f(x)$

   (a) $(x - \beta)$ (b) $(x - 2\beta)$

   (c) $(x + \beta)$ (d) $(2x - \beta)$

4. If $(y - a)$ is factor of $f(y)$ then ___ is a zero of $f(y)$

   (a) y (b) a

   (c) $2a$ (d) $2y$

5. Which of the following is not correct for : A quadratic polynomial may have

   (a) no real zeros (b) two equal real zeros

   (c) two distinct zeros (d) three real zeros.

6. Cubic poly $x = f(y)$ cuts y-axis at almost

   (a) one point (b) two points

   (c) three points (d) four points

7. Polynomial $x^2 + 1$ has ___ zeros

   (a) only one real (b) no real

   (c) only two real (d) one real and the other non-real.

8. If α, β are the zeros of the polynomials $f(x) = x^2 + x + 1$ then $\frac{1}{\alpha} + \frac{1}{\beta} =$ _____

   (a) 1 (b) –1

   (c) 0 (d) none

9. If one of the zero of the polynomial $g(x) = (k^2 + 4)\ x^2 + 13x + 4k$ is reciprocal of the other then k = ___

(a) 2 (b) – 2

(c) 1 (d) – 1

10. If 2 is a zero of both the polynomial, $3x^2 + ax - 14$ and $2x - b$ then $a - 2b$ = ___

(a) –2 (b) 7

(c) –8 (d) –7

11. If zeros of the polynomial $ax^2 + bx + c$ are reciprocal of each other then

(a) $a = c$ (b) $a = b$

(c) $b = c$ (d) $a = -c$

12. The zeros of the polynomial $h(x) = (x - 5)\ (x^2 - x - 6)$ are

(a) –2, 3, 5 (b) –2, –3, –5

(c) 2, –3, –5 (d) 2, 3, 5

13. Graph of $y = ax^2 + bx + c$ intersects x-axis at 2 distinct points if

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac < 0$

(c) $b^2 - 4ac = 0$ (d) none

SHORT ANSWER TYPE QUESTIONS

14. If α and β are the zeros of the polynomial $2x^2 - 7x + 3$. Find the sum of the reciprocal of its zeros.

15. If α, β are the zeros of the polynomial $p(x) = x^2 - a\ (x + 1) - b$ such that $(\alpha + 1)\ (\beta + 1) = 0$ then find value of b.

16. If α, β are the zeros of the polynomial $x^2 - (k + 6)\ x + 2\ (2k - 1)$. Find k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

17. If $(x + p)$ is a factor of the polynomial $2x^2 + 2px + 5x + 10$ find p.

18. Find a quadratic polynomial whose zeroes are $\left(5 - 3\sqrt{2}\right)$ and $\left(5 + 3\sqrt{2}\right)$.

19. If $\frac{1}{5}$ and -2 are respectively product and sum of the zeroes of a quadratic polynomial. Find the polynomial.

20. Find zeroes of $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

21. If $(x + k)$ is a factor of the polynomial $x^2-2x-15$ and $x^3 + a$. Find k and a.

22. Form a quadratic polynomial, one of whose zero is $\left(2 + \sqrt{5}\right)$ and the sum of zeros is 4.

23. If sum of the zeroes of $kx^2 + 3k + 2x$ is equal to their product. Find k.

24. If one zero of $4x^2 - 9 - 8kx$ is negative of the other find k.

LONG ANSWER TYPE QUESTIONS

25. Find the zeroes of $6x^2 - 3 - 7x$. Verify the relationship between the zeros and coefficients.

26. If one zero of he polynomial $(a^2 + a)\ x^2 + 13x + 6a$ is reciprocal of the other, find value (s) of a.

27. -5 is one of the zeroes of $2x^2 + px - 15$. Quadratic polynomial $p(x^2 + x) + k$ has both the zeros equal to each other. Then find k.

28. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of the zeros as half of their product.

29. If $f(x) = 2x^4 - 5x^3 + x^2 + 3x - 2$ is divided by $g(x)$ the quotient is $q(x) = 2x^2 - 5x + 3$ and $r(x) = -2x + 1$ find $g(x)$.

30. If $(x - 2)$ is one of the factors of $x^3 - 3x^2 - 4x + 12$ find the other zeros.

31. If α and β are the zeros of he polynomial $x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

32. If α, β are zeros of quadratic polynomial $2x^2 + 5x + k$, find the value of k, such that $(\alpha + \beta)^2 - \alpha\beta = 24$.

33. Obtain all zeros of $x^4 - x^3 - 7x^2 + x + 6$ if 3 and 1 are zeros.

34. Find all the zeros of the polynomial $4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeros are 2 and 3.

35. If $\left(2+\sqrt{3}\right)$ and $\left(2-\sqrt{3}\right)$ are two zeroes of $x^4 - 4x^3 - 8x^2 + 36x - 9$ find the other two zeroes.

36. What must be subtracted from $8x^4 + 14x^3 - 4x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

37. When we add $p(x)$ to $4x^4 + 2x^3 - 2x^2 + x - 1$ the resulting polynomial is divisible by $x^2 + 2x - 3$ find $p(x)$.

38. Find a and f if $(x^4 + x^3 + 8x^2 + ax + f)$ is a multiple of $(x^2 + 1)$.

39. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by $3x^2 + 1 + 4x$ then $r(x) = (ax + b)$ find a and b.

40. Obtain all the zeroes of $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if $\left(x \pm \sqrt{\frac{3}{2}}\right)$ are two factors of this polynomial.

41. Find all the zeroes of $x^4 - 3x^3 - x^2 + 9x - 6$ if $-\sqrt{3}$ and $\sqrt{3}$ are two of its zeros.

42. If $(x^3 - 3x + 1)$ is one of the factors of the polynomial $x^5 - 4x^3 + x^2 + 3x + 1$, find the other two factors.

43. What does the graph of the polynomial $ax^2 + bx + c$ represents. What type of graph will it represent (*i*) for a > 0, (*ii*) for a < 0. What happens if $a = 0$.

ANSWERS

1.	*b*	2.	*a*
3.	*a*	4.	*b*
5.	*a*	6.	*c*
7.	*b*	8.	*b*
9.	*a*	10.	*d*
11.	*a*	12.	*a*

13. a

14. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7}{3}$

15. 1

16. $k = 7$

17. $p = 2$

18. $x^2 - 10x + 7$

19. $x^2 + 2x + \frac{1}{5}$

20. $2\sqrt{3}, \frac{2}{3}\sqrt{3}$

21. $k = -5, 3$ and $a = -125 + 27$

22. $x^2 - 4x - 1$

23. $-\frac{2}{3}$

24. 0

25. $-\frac{1}{3}, \frac{3}{2}$

26. 5

27. $p = 7, k = \frac{7}{4}$

28. $k = 1$

29. $g(x) = x^2 - 1$

30. $-2, 3$

31. $k = 6$

32. $k = 2$

33. $-2, -1$

34. $-\frac{1}{2}, +\frac{1}{2}$

35. ± 3

36. $14x - 10$

37. $61x + 65$

38. $r(x) = 0$

$\Rightarrow (a-1)x + (f-7) = 0$

$\Rightarrow a = 1$ and $f = 7$

39. $r(x) = x + 2 = ax + f \Rightarrow a = 1$ and $f = 2$

40. $2, -1 \pm \sqrt{\frac{3}{2}}$

41. $\pm\sqrt{3}, 1, 2$

42. $(x-1), (x+1)$

43. A curve (parabola) upward parabola, downward parabola, straight line.