5. Prove that $\sqrt{2}$ is not a rational number.
6. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5, 7 and 11 respectively.
7. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ *is rational*.
8. Using prime factorization method, find the HCF and LCM of 72, 126 and 168. Also show that $HCF\ X\ LCM\ \neq product\ of\ the\ three\ numbers.$

2. Polynomials
(Key Points)

Polynomial:

An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \text{-----} + a_nx^n$ where $a_n \neq 0$ is called a polynomial in variable x of degree n. where; a_0, a_1, ----- a_n are real numbers and each power of x is a non negative integer.
Ex.:- $2x^2 - 5x + 1$ is a polynomial of degree 2.
Note: $\sqrt{x} + 3$ *is not a polynomial.*

- A polynomial $p(x) = ax + b$ *of degree* 1 *is called a linear polynomial.* Ex. 5x -3, 2x etc
- A polynomial $p(x) = ax^2 + bx + c$ *of degree* 2 *is called a quadratic polynomial.* Ex. $2x^2 + x - 1$, $1 - 5x + x^2$ etc.
- A polynomial $p(x) = ax^3 + bx^2 + cx + d$ *of degree* 3 *is called a cubic polynomial.* Ex. $\sqrt{3}x^3 - x + \sqrt{5}$, $x^3 - 1$ etc.

Zeroes of a polynomial: A real number k is called a zero of polynomial $p(x)$ *if* $p(x) = 0$. The graph of $y = p(x)$ intersects the X- axis.

- A linear polynomial has only one zero.
- A Quadratic polynomial has two zeroes.
- A Cubic polynomial has three zeroes.

For a quadratic polynomial: If α, β are zeroes of $P(x) = ax^2 + bx + c$ then :

1. Sum of zeroes = $\alpha + \beta = \frac{-b}{a} = \frac{-Coefficient\ of\ x}{coefficient\ of\ x^2}$
2. Product of zeroes = $\alpha \cdot \beta = \frac{c}{a} = \frac{Constant\ term}{coefficient\ of\ x^2}$

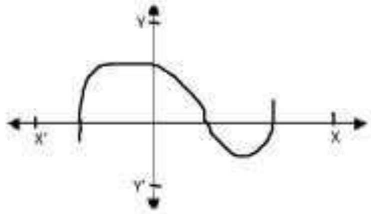
- A quadratic polynomial whose zeroes are α and β, is given by:
  $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$
  $= x^2 - (sum\ of\ zeroes)x + product\ of\ zeroes.$
- If α, β *and* γ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:
  - $\alpha + \beta + \gamma = \frac{-b}{a}$
  - $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
  - $\alpha\beta\gamma = \frac{-d}{a}$

Division algorithm for polynomials: If $p(x)$ *and* $g(x)$ *are any two polynomials with* $g(x) \neq 0$, then we can find polynomials $q(x)$ *and* $r(x)$ *such that:*

$p(x) = q(x)X\ g(x) + \ r(x), where\ r(x) = 0\ or\ degree\ of\ r(x) < \ degree\ of\ g(x).$

(Level - 1)

1. In a graph of $y = p(x), find\ the\ number\ of\ zeroes\ of\ p(x).$



Ans. 3.

2. If α, β are the zeroes of $f\ (x) = \ x^2 + x + 1$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$.

Ans. (-1)

3. Find a quadratic polynomial whose zeroes are $\frac{-2}{\sqrt{3}}\ and\ \frac{\sqrt{3}}{4}$.

Ans. $x^2 - \left(\frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4}\right)x + (-\frac{1}{2})$

4. If $p\ (x) = \ \frac{1}{3}x^2 - \ 5x + \frac{3}{2}$ then find its sum and product of zeroes.

Ans. Sum=15, Product = $\frac{9}{2}$

5. If the sum of zeroes of a given polynomial $f\ (x) = x^3 - 3kx^2 - x + 30$ is 6. Find the value of K.

Ans. $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{3k}{1} = 6$

$\therefore$ k = 2

6. Find the zero of polynomial $3x + 4$.

Ans. -4/3

7. Write the degree of zero polynomial.

Ans. Not defined.

(Level - 2)

1. Form a cubic polynomial with zeroes 3, 2 and -1.

Hints/Ans. $p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

2. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

Ans. Zeroes are 3/2 & -1/3.

3. For what value of k, (-4) is a zero of polynomial $x^2 - x - (2k + 2)$?

Ans. k=9

4. Give an example of polynomials $p(x), g(x), q(x) and\ r(x)\ which\ satisfy\ division\ algorithm\ and\ deg.\, p(x) = \ deg.\, g(x).$

Ans. $3x^2 + 2x + 1,\ x^2,\ 3,\ 2x + 1$

5. Find the zeroes of $4u^2 + 8u$.

Ans. 0, -2

6.Find a quadratic polynomial, whose the sum and product of its zeroes are $\frac{1}{4}, -1$.

Ans. $x^2 - \frac{1}{4}x - 1$

(Level - 3)

1. Find the zeroes of polynomial $x^3 - 2x^2 - x + 2$

Ans. -1, 1, 2

2. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $\alpha - \beta,\ \alpha,\ \alpha + \beta$. Find $\alpha\ and\ \beta$

Ans. $\alpha = 1,\ \beta = \pm\sqrt{2}$

3. Divide $f(x) = 6x^3 + 11x^2 - 39x - 65$ by $g(x) = x^2 - 1 + x$

Ans. Quotient=$6x + 5$; Remainder = $-38x - 60$

4. Check whether the polynomial $t^2 - 3$ is a factor of polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$ $by\ applying\ the\ division\ algorithm.$

Ans. Remainder=0, Quotient=$2t^2 + 3t + 4$, Given Polynomial is a factor.

(Level - 4)

1. Obtain all zeroes of $f(x) = x^3 + 13x^2 + 32x + 20$

Ans. -1, -2, -10

2. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans. -1 & -1

3. On dividing $x^3 - 3x^2 + x + 2\ by$ a polynomial $g(x)$,the quotient and remainder were $x - 2$ and $-2x + 4$ respectively, find $g(x)$.

Ans. $x^2 - x$+1

(PROBLEMS FOR SELF-EVALUATION)

1. Check whether $g(x) = 3x - 2$ is a factor of $p(x) = 3x^3 + x^2 - 20x + 12$.
2. Find quotient and remainder applying the division algorithm on dividing $p(x) = x^3 - 6x^2$+2x -4 by $g(x) = x - 1$.
3. Find zeros of the polynomial $2x^2 - 8x + 6$
4. Find the quadratic polynomial whose sum and product of its zeros are $\frac{2}{3}$, $\frac{-1}{3}$respectively.
5. Find the zeroes of polynomial $x^3 - 2x^2 - x + 2$
6. If one of the zeroes of the polynomial $2x^2 + px + 4 = 0\ is$ 2, find the other root, also find the value of p.
7. If $\alpha\ and\ \beta$ are the zeroes of the polynomial $kx^2 + 4x$+4 show that $\alpha^2 + \beta^2 = 24$, find the value of k.
8. If $\alpha\ and\ \beta$ are the zeroes of the equation $6x^2 + x - 2 = 0, find\ \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

---xxx---