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## Polynomial

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An expression of the form $\mathrm{p}(\mathrm{x})=a_{0}+a_{1} x+a_{2} x^{2}+\cdots \ldots \ldots \ldots+a_{n} x^{n}$ where $a_{n} \neq 0$ is called a polynomial in one variable $x$ of degree $n$, where; $a_{0}, a_{1}, a_{2} \ldots \ldots \ldots \ldots \ldots a_{n}$ are constants and they are called the coefficients of $x_{0}, x, x^{2}$ $x^{n}$. Each power of x is a non-negative integer.
Eg: $-2 x^{2}-5 x+1$ is a polynomial of degree 2
Note: $\sqrt{x}+3$ is not a polynomial

- A polynomial $\mathrm{p}(x)=a x+b$ of degree 1 is called a linear polynomial $\mathrm{Eg}: 5 x-3,2 x$ etc
- A polynomial $\mathrm{p}(x)=a x^{2}+b x+c$ of degree 2 is called a quadratic polynomial Eg : $2 x^{2}+x-1$
- A polynomial $p(x)=a x^{3}+b x^{2}+c x+d$ of degree 3 is called a cubic polynomial. Eg: $\sqrt{3} x^{3}-x+\sqrt{5}, x^{3}-1$ etc
Zeroes of a polynomial: A real number $k$ is called a zero of polynomial $p(x)$ if $p(k)=0$. If the graph of $y=p(x)$ intersects the $X$-axis at $n$ times, the number of zeroes of $y=p(x)$ is $n$.
- A linear polynomial has only one zero.
- A quadratic polynomial has two zeroes.
- A cubic polynomial has three zeroes.

Graphs of different types of polynomials:

- Linear polynomial:- The graph of a linear polynomial $a x+b$ is a straight line, intersecting $X$ - axis at one point

- Quadratic polynomial:-
(i) Graph of a quadratic polynomial $p(x)=a x^{2}+b x+c$ is a parabola open upwards like $U$, if $a>0$ \& intersects $x$-axis at maximum two distinct points.


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(ii) Graph of a quadratic polynomial $\mathrm{p}(\mathrm{x})=a x^{2}+b x+c$ is a parabola open downwards like $\cap$ if $a<0$ \& intersects $x$-axis at maximum two distinct points


- Cubic polynomial and its graph:- in general a polynomial $p(x)$ of degree $n$ crosses the $x$ axis at most $n$ points.


For a quadratic polynomial:- If $\alpha, \beta$ are zeroes of $p(x)=a x^{2}+b x+c$ then,

1. Sum of zeroes $=\alpha+\beta=-\frac{b}{a}=\frac{\text { coefficients of } x}{\text { coefficient of } x^{2}}$
2. Product of zeroes $=\alpha . \beta=\frac{c}{a}=\frac{\text { constant term }}{\text { coefficient of } x^{2}}$

- A quadratic polynomial whose zeroes are $\alpha$ and $\beta$, is given by:

$$
p(x)=x^{2}-(\alpha+\beta) x+\alpha \beta
$$

- If $\alpha, \beta$ and $\gamma$ are zeroes of the cubic polynomial $a x^{3}+b x^{2}+c x+d$ then:


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$$
\begin{gathered}
\alpha+\beta+\gamma=-\frac{b}{a} \\
\alpha \beta+\beta \gamma+\gamma \alpha=\frac{c}{a} \\
\alpha \beta \gamma=\frac{-d}{a}
\end{gathered}
$$

- If $\alpha, \beta \& \gamma$ are zeroes of a cubic polynomial $\mathrm{p}(\mathrm{x})$,

$$
p(x)=x^{3}-(\alpha+\beta+\gamma) x^{2}+(\alpha \beta+\beta \gamma+\gamma \alpha) x-\alpha \beta \gamma
$$

Division algorithm for polynomials: If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we have polynomials $q(x)$ and $r(x)$ such that
$\mathrm{P}(\mathrm{x})=\mathrm{g}(\mathrm{x}) \times q(x)+r(x), \quad$ where $\mathrm{r}(\mathrm{x})=0$ or degree of $\mathrm{r}(\mathrm{x})<$ degree of $\mathrm{g}(\mathrm{x})$.

## Nature of graph of polynomial $P(x)=a x^{2}+b x+c$ :-

Case-1 When polynomial $a x^{2}+b x+c$ is factorable in two distinct linear factors.

In this case, curve cuts $X$ - axis at two distinct points. The co-ordinate of the vertex of parabola are(-b/2a, -D/2a) where $\mathrm{D}=b^{2}-4 a c$. The x co-ordinates of these points are the two zeroes of the polynomial.


Case 2:- When Polynomial $a x^{2}+b x+c$ is factorisable into two equal factors.

In this case, curve touches $X$-axis at the point (-b/2a, 0). The $x$ - Co-ordinates of the point gives two equal zeroes of the polynomial.

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(i) $a>0$

(ii) $\mathrm{a}<0$

Case- 3 When Polynomial $a x^{2}+b x+c$ is not factorizable. In this case, the curve doesn't cut or touches $X$-axis


Y

(ii) $a<0$

## Level -I

1. Find the value of zeroes of the polynomials $p(x)$ as shown in the graph and hence find the polynomial.(CBSE 2014-15).

2. Let $\alpha$ and $\beta$ are the zeroes of a quadratic polynomial $2 x^{2}-5 x-6$ then form a quadratic polynomial whose zeroes are $\alpha+\beta$ and $\alpha \beta$. (CBSE 2011)

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3. Check whether $x^{2}+3 x+1$ is a factor of $3 x^{4}+5 x^{3}-7 x^{2}+2 x+2$ ? (CBSE 2010)
4. Can ( $x-7$ ) be the remainder on division of a polynomial $p(x)$ by $(7 x+2)$ ? Justify your answer(CBSE 2010)
5. What must be subtracted from the polynomial $f(x)=x^{4}+2 x^{3}-13 x^{2}-12 x+21$, so that the resulting polynomial is exactly divisible by $x^{2}-4 x+3$ ? (CBSE 2013)
6. Write the degree of zero polynomial?
7. Find the zeroes of a quadratic polynomial $6 x^{2}-7 x-3$ and verify the relationship between the zeroes and the coefficients? (CBSE 2014-15
8. Find the quadratic polynomial sum of whose zeroes is 2 V 3 and their product is 2 ?(CBSE 2008)

## Level II

9. If the sum of squares of the zeroes of the polynomials $6 x^{2}+x+k$ is $\frac{25}{36}$. find the value of $k$ ?( CBSE 2014-15)
10. If one zero of the quadratic polynomial $\mathrm{f}(\mathrm{x})=4 x^{2}-8 k x-9$ is negative of the other, then find the value of $k$ ?(CBSE 2014-15)
11. Find the values of k for which the quadratic equation $9 x^{2}-3 k x+k=0$ has equal roots. (CBSE 2014)
12.On dividing $3 x^{3}-2 x^{2}+5 x+5$ by the polynomial $\mathrm{p}(\mathrm{x})$, the quotient and remainder are $x^{2}-x+2$ and -7 respectively. Find $\mathrm{p}(\mathrm{x})$ ?(CBSE 2013)
12. Find all the zeroes of the polynomial $x^{4}+x^{3}-9 x^{2}-3 x+18$, if two of its zeroes are $\sqrt{3}$ and $\sqrt{-3}$. (CBSE 2010,13)
13. If $\alpha, \beta$ are zeroes of the quadratic polynomial $p(x)=x^{2}-(k-6) x+(2 k+1)$. Find the value of k if $\alpha+\beta=\alpha \beta$. (CBSE 2010)
15.If the zeroes of the polynomial $x^{2}-5 x+k$ are the reciprocal of each other, then find the value of $K$ ? (CBSE 2011)
14. If $\alpha$ and $\beta$ are zeroes of the quadratic polynomial $x^{2}-6 x+a$, find the value of ${ }^{\prime} a^{\prime}$. If $3 \alpha+2 \beta=20$.(CBSE 2010)

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## LEVEL III

17.On dividing $3 x^{3}+4 x^{2}+5 x-13$ by a polynomial $\mathrm{g}(\mathrm{x})$, the quotient and remainder are $3 x+10$ and $16 x-43$ respectively. Find the polynomial $\mathrm{g}(\mathrm{x})$.
18.If -5 is a root of quadratic equation $2 x^{2}+p x-15=0$ and the quadratic equation $p\left(x^{2}+x\right) k=0$ has equal roots, find the value of k .
(CBSE 2106)
19.If $\alpha, \beta$ and $\gamma$ are zeroes of the polynomial $6 x^{3}+3 x^{2}-5 x+1$, then find the values of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$.
(CBSE 2010)
20. Form a cubic polynomial whose zeroes are 3,2 and -1 . Hence find
(i) Sum of its zeroes
(ii) Sum of the product, taken two at a time
(iii) Product of its zero.

## (SELF EVALUATION QUESTIONS)

21.Find the number of zeroes of $p(x)$ in each case, for some polynomials $p(x)$.

22.If $\alpha$ and $\beta$ are the zeroes of the equation $6 x^{2}+x-2=0$, find $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$
23.If one of the zeroes of the polynomial $2 x^{2}+p x+4=0$ is 2 , find the other zero, also find the value of $p$
24.If one zero of the polynomial $\left(a^{2}+9\right) x^{2}+13 x+6 a$ is reciprocal of the other. Find the value of a. (All India)

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25.If $\alpha$ be the number of person who take junk food, $\beta$ be the person who take food at home and $\alpha$ and $\beta$ be the zeroes of quadratic polynomial $f(x)=x^{2}-3 x+2$, then find a quadratic polynomial whose zeroes are $\frac{1}{2 \alpha+\beta}$ and $\frac{1}{2 \beta+\alpha}$, which way of taking food you prefer and why?
26.If the number of apples and mangoes are the zeroes of the polynomial $3 x^{2}=8 x-$ $2 k+1$ and the number of apples is 7 times the number of mangoes, then find the number of zeroes and value of $k$. What are benefits of fruits in our daily life?

