NUMBER SYSTEMS

Numbers are intellectual witnesses that belong only to mankind.

1. If the H C F of 657 and 963 is expressible in the form of 657x + 963**x** - 15 find x. (Ans:x=22)

Ans: Using Euclid's Division Lemma
$$a= bq+r , o \leq r < b$$
$$963=657\times1+306$$
$$657=306\times2+45$$
$$306=45\times6+36$$
$$45=36\times1+9$$
$$36=9\times4+0$$
$\therefore$ HCF (657, 963) = 9

now $9 = 657x + 963\times (-15)$
$$657x=9+963\times15$$
$$=9+14445$$
$$657x=14454$$
$$x=14454/657$$

$$\boxed{x =22}$$

2. Express the GCD of 48 and 18 as a linear combination. (Ans: Not unique)

A=bq+r, where $o \leq r < b$
$$48=18x2+12$$
$$18=12x1+6$$
$$12=6x2+0$$
$\therefore$ HCF (18,48) = 6
now $6= 18-12x1$
$$6= 18-(48-18x2)$$
$$6= 18-48x1+18x2$$
$$6= 18x3-48x1$$
$$6= 18x3+48x(-1)$$
i.e. $6= 18x +48y$

$\therefore$ $\boxed{x=3 , y=-1}$

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$6= 18\times3 +48\times(-1)$
$=18\times3 +48\times(-1) + 18\times48-18\times48$
$=18(3+48)+48(-1-18)$
$=18\times51+48\times(-19)$
$6=18x+48y$

$\therefore$ $\boxed{x = 51, \ y = -19}$

Hence, x and y are not unique.

3. Prove that one of every three consecutive integers is divisible by 3.

Ans:
   n,n+1,n+2 be three consecutive positive integers
   We know that n is of the form 3q, 3q +1, 3q + 2
   So we have the following cases

   Case – I   when n = 3q

   In the this case, n is divisible by 3 but n + 1 and n + 2 are not divisible by 3

   Case - II   When n = 3q + 1
               Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

   Case – III When n = 3q +2
               Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

   Hence one of n, n + 1 and n + 2 is divisible by 3

4. Find the largest possible positive integer that will divide 398, 436, and 542 leaving remainder 7, 11, 15 respectively.
   (Ans: 17)

Ans: The required number is the HCF of the numbers

      Find the HCF of 391, 425 and 527 by Euclid's algorithm

      $\therefore$ HCF (425, 391) = 17

      Now we have to find the HCF of 17 and 527
          527 = 17 x 31 +0

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$$\therefore \text{ HCF }(17,527) = 17$$
$$\therefore \text{ HCF }(391, 425 \text{ and } 527) = 17$$

5. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

(Ans:2520)

Ans: The required number is the LCM of 1,2,3,4,5,6,7,8,9,10

$$\therefore \text{ LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 2520$$

6. Show that 571 is a prime number.

Ans: Let $x=571 \Rightarrow \sqrt{x}=\sqrt{571}$

Now 571 lies between the perfect squares of $(23)^2$ and $(24)^2$
Prime numbers less than 24 are 2,3,5,7,11,13,17,19,23
Since 571 is not divisible by any of the above numbers
571 is a prime number

7. If d is the HCF of 30, 72, find the value of x & y satisfying d = 30x + 72y.

(Ans:5, -2 (Not unique)

Ans: Using Euclid's algorithm, the HCF (30, 72)

$72 = 30 \times 2 + 12$
$30 = 12 \times 2 + 6$
$12 = 6 \times 2 + 0$

HCF (30,72) = 6
$6=30-12 \times 2$
$6=30-(72-30 \times 2)2$
$6=30-2 \times 72+30 \times 4$
$6=30 \times 5+72 \times -2$
$\therefore x = 5, y = -2$

Also $6 = 30 \times 5 + 72 (-2) + 30 \times 72 - 30 \times 72$

Solve it, to get

$$x = 77, y = -32$$

Hence, x and y are not unique

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8. Show that the product of 3 consecutive positive integers is divisible by 6.

Ans: Proceed as in question sum no. 3

9. Show that for odd positive integer to be a perfect square, it should be of the form 8k +1.

Let a=2m+1

Ans: Squaring both sides we get

$$a^2 = 4m \, (m +1) + 1$$

∴ product of two consecutive numbers is always even

$m(m+1)=2k$

$a^2=4(2k)+1$

$a^2 = 8 \, k + 1$

Hence proved

10. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36. (Ans:999720)

Ans: LCM of 24, 15, 36

$$LCM = 3 \times 2 \times 2 \times 2 \times 3 \times 5 = 360$$

Now, the greatest six digit number is 999999
Divide 999999 by 360
∴ Q = 2777 , R = 279

∴ the required number = 999999 – 279 = 999720

11. If a and b are positive integers. Show that √2 always lies between $\dfrac{a}{b} \, and \, \dfrac{a-2b}{a+b}$

Ans: We do not know whether $\dfrac{a^2-2b^2}{b(a+b)}$ or $\dfrac{a}{b} < \dfrac{a+2b}{a+b}$

∴ to compare these two number,

Let us comute $\dfrac{a}{b} - \dfrac{a+2b}{a+b}$

=> on simplifying , we get $\dfrac{a^2-2b^2}{b(a+b)}$

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$$\therefore \quad \frac{a}{b} - \frac{a+2b}{a+b} > 0 \text{ or } \frac{a}{b} - \frac{a+2b}{a+b} < 0$$

now $\dfrac{a}{b} - \dfrac{a+2b}{a+b} > 0$

$\dfrac{a^2 - 2b^2}{b(a+b)} > 0$ solve it , we get , $a > \sqrt{2}b$

Thus   , when $a > \sqrt{2}b$ and
$$\frac{a}{b} < \frac{a+2b}{a+b},$$

We have to prove that $\dfrac{a+2b}{a+b} < \sqrt{2} < \dfrac{a}{b}$

Now $a > \sqrt{2}\,b \Rightarrow 2a^2 + 2b^2 > 2b^2 + a^2 + 2b^2$
On simplifying   we get

$$\sqrt{2} > \frac{a+2b}{a+b}$$

Also $a > \sqrt{2}$

$\Rightarrow \dfrac{a}{b} > \sqrt{2}$

Similarly we get $\sqrt{2}, < \dfrac{a+2b}{a+b}$

Hence $\dfrac{a}{b} < \sqrt{2} < \dfrac{a+2b}{a+b}$

12. Prove that $(\sqrt{n-1} + \sqrt{n+1})$ is irrational, for every $n \in N$

Self Practice

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