UNIT-7

CO-ORDINATE GEOMETRY

Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field.

1. Find the points on the y axis whose distances from the points (6, 7) and (4,-3) are in the ratio 1:2. [Ans:(0, 9) , (0, $\frac{35}{3}$)]

Ans: Point on y-axis (0, y)
A(6, 7) B(4, -3) ratio 1:2

$$\frac{6^2+(7-y)^2}{4^2+(-3-y)^2}=\left(\frac{1}{2}\right)^2$$

On solving we get (0, 9) & (0, $\frac{35}{3}$)

2. Determine the ratio in which the line 2x + y -4 = 0 divide the line segment joining the points A (2,-2) and B (3, 7).Also find the coordinates of the point of division. [Ans:2 : 9 , ($\frac{24}{11}$,-$\frac{4}{11}$)]

Ans : Let the ratio be k:1
Let the co-ordinates of point of division be (x, y)

$$\therefore x=\frac{k(3)+1.2}{k+1} \qquad =\frac{3k+2}{k+1}$$

$$y=\frac{k(7)-1.2}{k+1} \qquad =\frac{7k-2}{k+1}$$

(x, y) lies on the line 2x + y – 4 = 0.

$$\therefore\ 2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

2(3k+2) + (7k-2) – 4 (k+1) = 0
6k + 4 + 7k - 2 – 4k – 4 = 0
9k – 2 = 0 k = $\frac{2}{9}$
Ratio is 2:9

$$\therefore x = \frac{3x\frac{2}{9}+2}{\frac{2}{9}+1} = \frac{\frac{2}{3}+2}{\frac{11}{9}} = \frac{\frac{2+6}{3}}{\frac{11}{9}} = \frac{8}{3} x \frac{9}{11} = \frac{24}{11}$$

$$y = \frac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1} = \frac{\frac{14-18}{9}}{\frac{11}{9}} = \frac{-4}{9} x \frac{9}{11} = \frac{-4}{11}$$

$\therefore$(x, y) =($\frac{24}{11}$, $\frac{-4}{11}$)

3. Find the third vertex of a triangle if its two vertices are (-1, 4) and (5, 2) and mid point of one side is (0, 3). (Ans: (-5, 4) or (1, 2))

Ans : Let the third vertex be (x, y)

If (0,3) is mid point of BC then

$\frac{x+5}{2}$=0 (or) x = - 5

$\frac{y+2}{2}$=3 y=4. (-5, 4)

If (0,3) is mid point of AC then

$\frac{x-1}{2}$=0 x=1 $\frac{y+4}{2}$=3 y + 4 = 6 y = 2 (1, 2)

$\therefore$ (-5. 4) or (1, 2) are possible answers.

4. If the vertices of a triangle are (1, k), (4, -3), (-9, 7) and its area is 15 sq units, find the value(s) of k.. $\left[\text{Ans: -3 , } \frac{21}{13}\right]$

Ans: A(1, k) B(4, -3) C(-9, 7)

Area of Δ ABC = $\frac{1}{2}$ [x_1 (y_2-y_3)+x_2(y_3-y_1) + x_1(y_1-y_2)]

=$\frac{1}{2}$ [1(-3-7)+4(7-k)+(-9)(k+3)] = 15

-10 + 28 – 4k – 9k – 27 = 30

- 9 – 13k = 30 $\Rightarrow$ k = -3

| -9 – 13k | = 30

9 + 13k = 30

k = $\frac{21}{13}$

k = -3, $\frac{21}{13}$

5. The centre of a circle is (2x – 1, 3x + 1).Find x if the circle passes through (-3,-1) and the length of the diameter is 20 units. $\left[\text{Ans: } x = 2, -\frac{46}{13}\right]$

Ans : D = 20 R = 10

$(2x - 1 + 3)^2 + (3x + 1 + 1)^2 = 10^2$

$(2x + 2)^2 + (3x + 2)^2 = 100$

$4x^2 + 8x + 4 + 9x^2 + 12x + 4 = 100$

$13x^2 + 20x + 8 = 100$

$13x^2 + 20x - 92 = 0$

$13x^2 + 46x - 26x - 92 = 0$

$(13x + 46) - 2(13x + 46) = 0$

$x = 2, \frac{-46}{13}$

6. If A & B are (-2,-2) and (2,-4) respectively, find the co ordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB. $\left[\text{Ans: } (-\frac{2}{7}, -\frac{20}{7})\right]$

Ans : $AP = \frac{3}{7}AB$

$\frac{AP}{AB} = \frac{3}{7}$ (i.e) $\frac{AP}{PB} = \frac{3}{4}$

AB = AP + PB

AP : PB = 3:4

Let P(x, y)

$x = \frac{3(2) + 4(-2)}{7} = \frac{6-8}{7} = \frac{-2}{7}$

$y = \frac{3(-4) + 4(-2)}{7} = \frac{-12-8}{7} = \frac{-20}{7}$

$(x, y) = (\frac{-2}{7}, \frac{-20}{7})$

7. Show that the points (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order are the vertices of a rhombus. Also find the area of the rhombus. (Ans: 24 sq units)

Ans : Let AC be d_1 & BD be d_2

Area = $\frac{1}{2}d_1d_2$

$d_1 = \sqrt{(3+1)^2 + (0-4)^2} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$d_2 = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

Area $= \frac{1}{2} d_1 d_2 = \frac{1}{2}$ x 4 x 6 x 2 = 24sq units.

8. If A, B and P are the points (-4, 3), (0, -2) and (α,β) respectively and P is equidistant from A and B, show that $8\alpha - 10\beta + 21 = 0$.

Ans : $AP = PB \Rightarrow AP^2 = PB^2$

$(\alpha + 4)^2 + (\beta - 3)^2 = \alpha^2 + (\beta + 2)^2$

$\alpha^2 + 8\alpha + 16 + \beta^2 - 6\beta + 9 = \alpha^2 + \beta^2 + 4\beta + 4$

$8\alpha - 6\beta - 4\beta + 25 - 4 = 0$

$8\alpha - 10\beta + 21 = 0$

9. If the points (5, 4) and (x, y) are equidistant from the point (4, 5), prove that $x^2 + y^2 - 8x - 10y + 39 = 0$.

Ans : AP = PB

$AP^2 = PB^2$

$(5-4)^2 + (4-5)^2 = (x-4)^2 + (y-5)^2$

$1 + 1 = x^2 - 8x + 16 + y^2 - 10y + 25$

$x^2 + y^2 - 8x - 10y + 41 - 2 = 0$

$x^2 + y^2 - 8x - 10y + 39 = 0$

10. If two vertices of an equilateral triangle are (0, 0) and (3, 0), find the third vertex.
.

$\left[\text{Ans: } \frac{3}{2}, \frac{3\sqrt{3}}{2} \text{ or } \frac{3}{2}, -\frac{3\sqrt{3}}{2}\right]$

Ans: OA = OB = AB

$OA^2 = OB^2 = AB^2$

$OA^2 = (3-0)^2 + 0 = 9$

$OB^2 = x^2 + y^2$

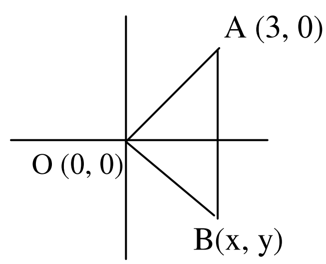
$AB^2 = (x-3)^2 + y^2 = x^2 + y^2 - 6x + 9$

$OA^2 = OB^2 = AB^2$

$OA^2 = OB^2$ & $OB^2 = AB^2$

$9 = x^2 + y^2 \Rightarrow y^2 = 9 - x^2$

$x^2 + y^2 - 6x + 9 = 9$



$x^2 + 9 - x^2 - 6x + 9 = 9$

$6x = 9 \qquad x = 3/2$

$y^2 = 9 - \left(\frac{3}{2}\right)^2 = 9 - \frac{9}{4} = \frac{36-9}{4} = \frac{27}{4}$

$y = \pm \frac{3\sqrt{3}}{2}$

$\therefore$ Third vertex is $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ or $\left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

11. Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3). Also find the radius. (Ans: (3, -2), 5 units)

Ans: OA=OB = OC = radius of the circle where O is the centre of the circle and let O be (x, y)

$OA^2 = OB^2 = OC^2$

$OA^2 = (x-6)^2 + (y+6)^2 = x^2 + y^2 - 12x + 36 + 12y + 36$

$OB^2 = (x-3)^2 + (y+7)^2 = x^2 + y^2 - 6x + 9 + 14y + 49$

$OC^2 = (x-3)^2 + (y-3)^2 = x^2 + y^2 - 6x + 9 - 6y + 9$

$OA^2 = OB^2$

$x^2 + y^2 - 12x + 12y + 72 = x^2 + y^2 - 6x + 14y + 58$

$-12x + 12y + 6x - 14y + 72 - 58 = 0$

$-6x - 2y + 14 = 0$

$-3x - y + 7 = 0$ ....(1)

$x^2 + y^2 - 6x + 9 + 14y + 49 = x^2 + y^2 - 6x + 9 - 6y + 9$

$-6x + 14y + 58 = -6x - 6y + 18$

$14y + 6y = 18 - 58$

$20y = -40$

$y = -2$ ....(2)

Substituting we get

$-3x + 2 + 7 = 0$

$-3x = -9$

$x = 3$

$(x, y) = (3, -2)$

Diameter $= 3^2 + 2^2 - 6(3) + 18 - 6(-2)$

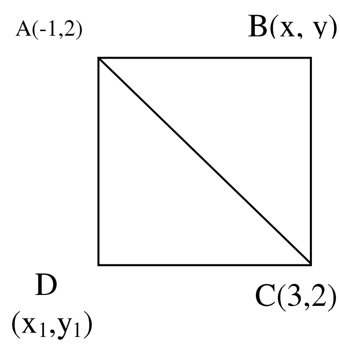
$= 9 + 4 - 18 + 18 + 12$

$= 13 + 12 = 25$

Radius $= \sqrt{25} = 5$ units

12. The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices. (Ans: (1, 0), (1, 4))

Ans : $AB = BC \Rightarrow AD^2 = BC^2$



$(x + 1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 - 4y + 4$

$2x - 4y + 5 = -6x - 4y + 13$

$8x = 13 - 5$ $8x = 8 \Rightarrow x = 1$

On substituting in $(x-3)^2 + (y-2)^2 + (x+1)^2 + (y-2)^2$

$= (-1 -3)^2 + (2 - 2)^2$
We get y = 4 or 0.
$\therefore$B (1, 4) or (1, 0)

AD = DC $\Rightarrow$ $AD^2 = DC^2$
$(x_1 + 1)^2 + (y_1-2)^2 = (x_1-3)^2 + (y_1-2)^2$
$\therefore$ x = 1.
On substituting in $(x_1 + 1)^2 + (y_1-2)^2 + (x_1-3)^2 + (y_1-2)^2 = 16$
We get $y_1 = 0$ or 4.
$\therefore$ D (1, 4) or (1, 0)
$\therefore$the opposite vertices are (1, 4) & (1, 0)

13. Find the coordinates of the point P which is three –fourth of the way from A (3, 1) to B (-2, 5). (Ans: $\left(-\frac{3}{4}, 4\right)$

Ans : Hint: Ratio AP:PB = 3 : 1

14. The midpoint of the line joining (2a, 4) and (-2, 3b) is (1, 2a +1).Find the values of a & b.

(Ans: a = 2, b = 2)

Ans : A(2a, 4) P(1, 2a + 1) B(-2, 3b)

$\frac{2a-2}{2} = 1$ & $\frac{4+3b}{2} = 2a+1$

We get a = 2 & b = 2.

15. Find the distance between the points (b + c, c + a) and (c + a, a + b) .

(Ans : $\sqrt{a^2 + 2b^2 + c^2 - 2ab - 2bc}$)

Ans : Use distance formula

16. Find the relation between x and y when the point (x,y) lies on the straight line joining the points (2,-3) and (1,4) [Hint: Use area of triangle is 0]

Ans : Hint: If the points are on straight line, area of the triangle is zero.

17. Find the distance between (cosθ, sinθ) and (sinθ, -cosθ). (Ans: √2)

Ans : $\sqrt{(\cos\theta - Sin\theta)^2 + (Sin\theta + \cos\theta)^2}$

On simplifying we get $\sqrt{2}$

18. Find the distance between (a cos35°, 0) (0, a cos65°). (Ans: a)

Ans : Proceed as in sum no.17.

19. The vertices of a $\Delta\ ABC$ are A(4, 6), B(1. 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the Δ ADE and compare it with the area of Δ ABC. (Ans: $\frac{15}{32}$ sq units; 1:16)

Ans : Hint : $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$\therefore$ AD : DB = 1 : 3 & AE : EC = 1 : 3

Find D & E and find area of triangle ADE and triangle ABC and compare.

20. Plot the points A(2,0) and B (6,0) on a graph paper. Complete an equilateral triangle ABC such that the ordinate of C be a positive real number .Find the coordinates of C

(Ans: $(4,2\sqrt{3})$

Ans : Proceed by taking C(x, y)

AC = BC = AB

21. Find the ratio in which the line segment joining A(6,5) and B(4,-3) is divided by the line y=2 (Ans:3:5)

Ans : Let the ratio be k:1

$$x = \frac{4k+6}{k+1}$$

$$y = \frac{-3k+5}{k+1}$$

$$\frac{-3k+5}{k+1} = 2$$

On solving we get k = 3 : 5

22. The base BC of an equilateral triangle ABC lies on the y-axis. The coordinates of C are (0,-3). If the origin is the midpoint of BC find the coordinates of points A and B.

Ans : Hint : The point A will lie on the x axis. Find A using AB = BC = AC.

Coordinates of B (0, 3)