COORDINATE GEOMETRY

**LEVEL – I ( 1 mark)**

**Q 1) Find the distance between the pairs of points : (-5,7) , (-1,3)**

Sol. Let the given points be A(-5,7) and B(-1,3)

Using distance formula, we have

AB = $\sqrt{(-1+5)^2 + (3-7)^2}$

$=\sqrt{4^2 + (-4^2)}$

$=\sqrt{16+16}$

$=\sqrt{32} = 4\sqrt{2}$ units

**Q 2) Find the point on y-axis which is equidistant from the points (5,-2) and (-3, 2).**

Sol .We know that a point on the y axis is the form (0,y). So, let the points P(0,y) be equidistant from A (5,-2) and B(-3,2). Then

$(5-0)^2 + (-2-y)^2 = (-3-0)^2 + (2-y)^2$

$25+4+y^2 +4y = 9 + 4 + y^2 - 4y$

$8y = -16$

$y = -2$

Hence, the required point is (0,-2).

Checking :

AP = $\sqrt{(5-0)^2 + (-2+2)^2} = \sqrt{25+0} = \sqrt{25} = 5$

BP = $\sqrt{(-3-0)^2 + (-2-2)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

Q 3)**Two vertices of a Triangle are (3 , -5) and ( -7, 4) . If its centroid is ( -2 , 1), find the third vertex** .

Solution: Let the coordinates of the third vertex be ( x , y) . Then ,

$\frac{x+3-7}{3}$=2 and $\frac{y-5+4}{3}$= -1

x-4 = 6 and y-1= -3

x = 10 and y = -2

Hence, third vertex of triangle is ( 10 , -2)

Q4)**If the mid points of the line segment joining the points P ( 6, b-2) and Q ( -2, 4) is (2, -3),find the value of b .**

Solution: The coordinates of the mid-point of PQ are $[\frac{6-2}{2},\frac{b-2+4}{2}]$

i.e. $[2,\frac{b+2}{2}]$

Equating it to (2 , -3)

$[\frac{b+2}{2}]$ = -3

b= -8



**<u>LEVEL – II ( 2 marks)</u>**

**Q 5) The line joining the points (2,-1) and (5,-6) is bisected at P. If P lies on the line 2x + 4y + k = 0. Find the value of k.**

Sol. The coordinates of P are $[\frac{2+5}{2},\frac{-1-6}{2}]$, i.e, P$[\frac{7}{2},\frac{-7}{2}]$

Since P lies on the line 2x + 4y + k = 0

$2(\frac{7}{2}) + 4(\frac{-7}{2}) + k = 0$

7 – 14 + k = 0

k = 7

**Q 6) Find the co-ordinates of the point which divides the line segment joining the points(6, 3) and (-4, 5) in the ration 3:2 internally.**

3 : 2

A(6,3) P(x,y) B(-4,5)

Let P(x, y) divides the line segment joining A(6, 3) and B(-4, 5) in the ratio 3 :2

P (x,y) = P [ $\frac{3(-4)+2(6)}{3+2}$ , $\frac{3(5)+2(6)}{3+2}$ ]

= P [ $\frac{-12+12}{5}$ , $\frac{15+12}{5}$ ]

= P [ $\frac{0}{5}$ , $\frac{21}{5}$ ]

Therefore, the coordinates of the point P are( 0 , $\frac{21}{5}$ )

Q7)**In each of the following find the value of '*k*', for which the points are collinear. (7, − 2), (5, 1), (3, − *k*)**

Solution -: For collinear points, area of triangle formed by them is zero
Therefore,for points (7, −2) (5, 1), and (3, *k*), area = 0

$$\frac{1}{2}\left[7\{1-k\}+5\{k-(-2)\}+3\{(-2)-1\}\right]=0$$
$$7-7k+5k+10-9=0$$
$$-2k+8=0$$
$$k=4$$

Q8 )**Find the coordinates of the points of trisection of the line segment joining the points A(2 , -2) and B ( -7 , 4).**

Solution -: Let P and Q be the points of trisection of AB

Therefore, AP = PQ = QB

A P Q B

(2,-2) (-7,4)

P divides AB internally in the ratio 1:2.

So, the coordinates of P , by applying the section formula are

$$\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}$$

i.e. , (-1 , 0)

now, Q also divides AB internally in the ratio 2:1 . so, the coordinates of Q are

$$\frac{2(-7)+1(2)}{1+2}, \frac{2(4)+1(-2)}{1+2}$$

i.e. (-4 , 2)

**LEVEL – III ( 3 marks )**

**Q 9) If the vertices of a triangle are (1, k), (4, -3), (-9, 7) and its area is 15 sq units, find the value(s) of k.**

Sol. Let A(1, k) ,B(4, -3)and C(-9, 7) be the vértices of triangle

Area of $\Delta$ ABC = $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_1(y_1-y_2)]$

$= \frac{1}{2}[1(-3-7)+4(7-k)+(-9)(k+3)] = 15$

$-10 + 28 - 4k - 9k - 27 = 30s$

$-9 - 13k = 30$

$-13k = 30+9$

$k = \frac{39}{-13}$

$k = -3$

**10) Find the point on the *x*-axis which is equidistant from (2, − 5) and (− 2, 9).**

Sol. We have to find a point on *x*-axis. Therefore, its *y*-coordinate will be 0.
Let the point on *x*-axis be (x,0)

Distance between $(x,0)$ and $(2,-5) = \sqrt{(x-2)^2+(0-(-5))^2} = \sqrt{(x-2)^2+(5)^2}$

Distance between $(x,0)$ and $(-2,9) = \sqrt{(x-(-2))^2+(0-(-9))^2} = \sqrt{(x+2)^2+(9)^2}$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2+(5)^2} = \sqrt{(x+2)^2+(9)^2}$$
$$(x-2)^2+25=(x+2)^2+81$$
$$x^2+4-4x+25=x^2+4+4x+81$$
$$8x=25-81$$
$$8x=-56$$
$$x=-7$$

**11) Determine the ratio in which the point P(m, 6) divides the join of A( -4, 3) and B (2,8).**

A(-4,3) k P(m,6) 1 B(2,8)

Let required ratio = k:1

Using section formula $\left(\frac{mx_1+nx_2}{m+n}, \frac{my_1+ny_2}{m+n}\right)$

For y-coordinate $6 = \left(\frac{8k+3}{k+1}\right)$

6(k+1) = 8k + 3

6k + 6 = 8k +3

6k – 8k = 3 – 6

-2k = -3

k = 3/2

Therefore required ratio = 3:2

**12) Find the value of k so that the points A (-2,3), B (3,-1) and C (5,k) are collinear.**

Sol. Here, $x_1 = -2, x_2 = 3, x_3=5$ ; $y_1=3, y_2=-1, y_3=k$

Area of $\Delta ABC = \frac{1}{2}[\ x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\ ]$

$= \frac{1}{2}[\ -2(-1-k) + 3(k-3) + 5(3+1)]$

$= \frac{1}{2}[\ 2 + 2k + 3k - 9 + 20]$

$= \frac{1}{2}[\ 5k + 13\ ]$

Now, the three points will be collinear

If the area of $\Delta ABC = 0$, i.e, if $\frac{1}{2}[\ 5k + 13\ ] = 0$

$5k + 13 = 0$

$k = -\frac{13}{5}$



**LEVEL IV ( 4 marks)**

Q**13)Find the value of y for which the distance between the points P(2-3) and Q(10,y) is10 units.**

Sol. Given P(2,-3)and Q(10,y)

P ——— 10 units ——— Q

$PQ=10$

$PQ^2 = 10^2 = 100$

Using distance formula

$(10-2)^2+(y-(-3))^2=100$

$8^2+(y+3)^2 = 100$

$64+y^2+6y+9 = 100$

$y^2+6y-27 = 0$

$y^2+9y-3y-27 = 0$

y(y+9)-3(y+9) = 0

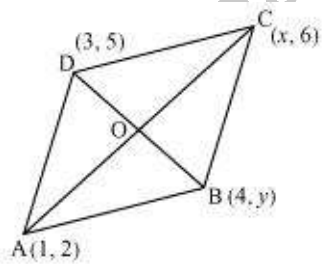
(y+9)(y-3) = 0

y+9=0 or y-3=0

Either y = -9 or y = 3

**Hence the required value of y can be -9 or 3**

**Q 14) If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.**




Let (1, 2), (4, *y*), (*x*, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals. Therefore, O is the mid-point of AC and BD.
If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2},\frac{2+6}{2}\right)\Rightarrow\left(\frac{x+1}{2},4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2},\frac{5+y}{2}\right)\Rightarrow\left(\frac{7}{2},\frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$
$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8$$
$$\Rightarrow x = 6 \text{ and } y = 3$$

**Q15.Do the points (3,2) ,(-2,-3) and (2,3) form a triangle? If so, name the type of triangle formed.**

Solution.-: Applying the distance formula to find the distances PQ, QR, and PR,where P(3,2) , Q(-2,-3) and R(2,3) then

PQ= $\sqrt{(-2-3)^2+(-3-2)^2}$

=$\sqrt{(-5)^2+(-5)^2}$ =$\sqrt{25+25}$ =$\sqrt{50}$

QR= $\sqrt{(2+2)^2+(3+3)^2}$

=$\sqrt{(4)^2+(6)^2}$ =$\sqrt{16+36}$ =$\sqrt{52}$

PR= $\sqrt{(2-3)^2+(3-2)^2}$

=$\sqrt{(-1)^2+(1)^2}$ =$\sqrt{1+1}$ =$\sqrt{2}$

Since the sum of any two of these distances is greater than the third distance, the points P,Q and R form a triangle.

Also, $PQ^2+PR^2= QR^2$

By the converse of Pythagoras Theorem , we have $\angle P = 90^0$

Therefore, PQR is a right triangle.

Q16.**Do the points (3,2) ,(-2,-3) and (2,3) form a triangle? If so, name the type of triangle formed.**

Solution.-: Applying the distance formula to find the distances PQ, QR, and PR,where P(3,2) , Q(-2,-3) and R(2,3) then

PQ= $\sqrt{(-2-3)^2+(-3-2)^2}$

$=\sqrt{(-5)^2+(-5)^2}$ $=\sqrt{25+25}$ $=\sqrt{50}$

QR= $\sqrt{(2+2)^2+(3+3)^2}$

$=\sqrt{(4)^2+(6)^2}$ $=\sqrt{16+36}$ $=\sqrt{52}$

PR= $\sqrt{(2-3)^2+(3-2)^2}$

$=\sqrt{(-1)^2+(1)^2}$ $=\sqrt{1+1}$ $=\sqrt{2}$

Since the sum of any two of these distances is greater than the third distance, the points P,Q and R form a triangle.

Also, $PQ^2+PR^2=QR^2$

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