

CIRCLES

1. Prove that the parallelogram circumscribing a circle is rhombus.

Ans Given : ABCD is a parallelogram circumscribing a circle.

To prove : - ABCD is a rhombus

or

$$AB=BC=CD=DA$$

Proof: Since the length of tangents from external are equal in length

$$\therefore AS = AR \quad \dots(1)$$

$$BQ = BR \quad \dots(2)$$

$$QC = PC \quad \dots(3)$$

Adding (1), (2), (3) & (4).

$$AS + SD + BQ + QC = AR + BR + PC + DP$$

$$AD + BC = AB + DC$$

$$AD + AD = AB + AB$$

Since $BC = AD$ & $DC = AB$ (opposite sides of a parallelogram are equal)

$$2AD = 2AB \quad \dots(4)$$

$$\therefore AD = AB \quad \dots(5)$$

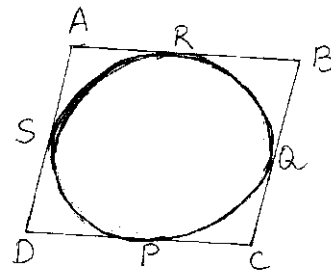
$BC = AD$ (opposite sides of a parallelogram)

$$DC = AB \quad \dots(6)$$

From (5) and (6)

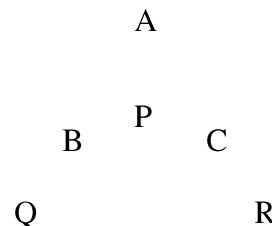
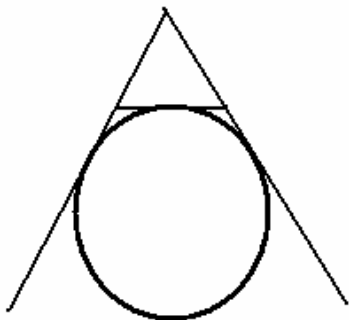
$$AB = BC = CD = DA$$

Hence proved



2. A circle touches the side BC of a triangle ABC at P and touches AB and AC when produced at Q and R respectively as shown in figure.

Show that $AQ = \frac{1}{2}$ (perimeter of triangle ABC)



Ans: Since the length of tangents from external point to a circle are equal.

$$AQ = AR$$

$$BQ = BP$$

$$PC = CR$$

$$\text{Since } AQ = AR$$

$$AB + BQ = AC + CR$$

$$\therefore AB + BP = AC + PC \text{ (Since } BQ = BP \text{ \& } PC = CR)$$

$$\text{Perimeter of } \Delta ABC = AB + AC + BC$$

$$= AB + BP + PC + AC$$

$$= AQ + PC + AC \text{ (Since } AB + BP = AQ)$$

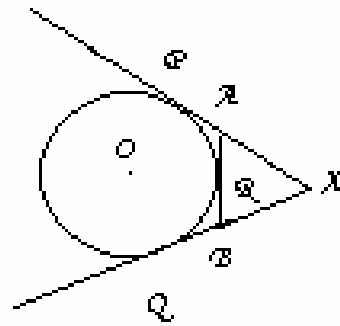
$$= AQ + AB + BP \text{ (Since } PC + AC = AB + BP)$$

$$= AQ + AQ \text{ (Since } AB + BP = AQ)$$

$$\text{Perimeter of } \Delta ABC = 2AQ$$

$$\therefore AQ = \frac{1}{2} (\text{perimeter of triangle } ABC)$$

3. In figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that $XA + AR = XB + BR$



Ans: Since the length of tangents from external point to a circle are equal

$$XP = XQ$$

$$PA = RA$$

$$BQ = BR$$

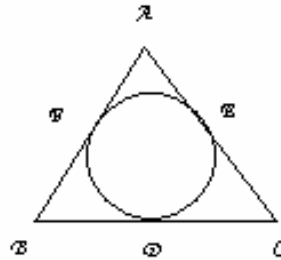
$$XP = XQ$$

$$\Rightarrow XA + PA = XB + BQ$$

$$\Rightarrow XA + AR = XB + BR \text{ (}\because PA = AR \text{ \& } BQ = BR)$$

Hence proved

4. In figure, the incircle of triangle ABC touches the sides BC, CA, and AB at D, E, and F respectively. Show that $AF+BD+CE=AE+BF+CD=\frac{1}{2}$ (perimeter of triangle ABC),



Ans: Since the length of tangents from an external point to are equal

$$\therefore AF = AE$$

$$FB = BD$$

$$EC = CD$$

$$\begin{aligned} \text{Perimeter of } \triangle ABC &= AB + BC + AC \\ &= AF + FB + BD + DC + AE + EC \\ &= AF + BD + BD + CE + AF + CE \\ &\quad (\because AF=AE, FB=BD, EC=CD) \\ &= AF + AF + BD + BD + CE + CE \end{aligned}$$

$$\text{Perimeter of } \triangle ABC = 2(AF + BD + CE)$$

$$\therefore AF + BD + CE = \frac{1}{2} (\text{perimeter of } \triangle ABC) \dots\dots\dots(1)$$

$$\begin{aligned} \text{Perimeter of } \triangle ABC &= AB + BC + AC \\ &= AF + FB + BD + DC + AE + EC \\ &= AE + BF + BF + CD + AE + CD \\ &\quad (\because AF = AE, FB = BD, EC = CD) \\ &= AE + AE + BF + BF + CD + CD \end{aligned}$$

$$\text{Perimeter of } \triangle ABC = 2(AE + BF + CD)$$

$$\therefore AE + BF + CD = \frac{1}{2} (\text{perimeter of } \triangle ABC) \dots\dots\dots(2)$$

From (1) and (2)

$$AF + BD + CE = AE + BF + CD = \frac{1}{2} (\text{perimeter of } \triangle ABC)$$

5. A circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

Ans: To prove :- $\angle AOB + \angle DOC = 180^\circ$
 $\angle BOC + \angle AOD = 180^\circ$

Proof : - Since the two tangents drawn from an external point to a circle subtend equal angles at centre.

$$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$$

$$\text{but } \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^\circ$$

$$\angle 2 + \angle 3 + \angle 6 + \angle 7 = 360^\circ$$

$$\therefore \angle AOB + \angle DOC = 180^\circ$$

Similarly

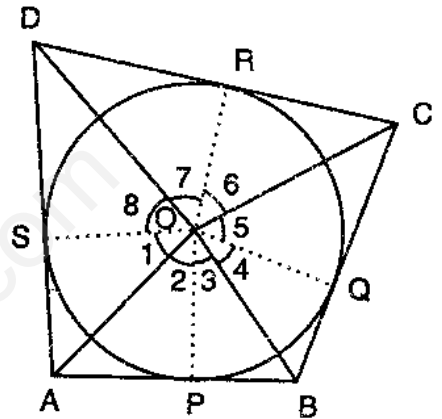
$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^\circ$$

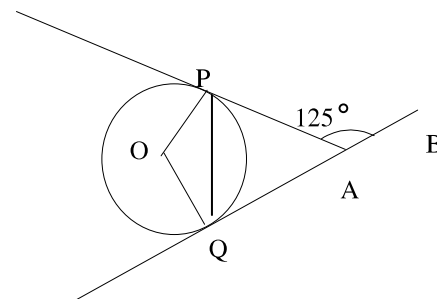
$$\angle 1 + \angle 8 + \angle 5 = 180^\circ$$

$$\therefore \angle BOC + \angle AOD = 180^\circ$$

Hence proved



6. In figure, O is the centre of the Circle .AP and AQ two tangents drawn to the circle. B is a point on the tangent QA and $\angle PAB = 125^\circ$, Find $\angle POQ$.
 (Ans: 125°)



Ans: Given $\angle PAB = 125^\circ$

To find : - $\angle POQ = ?$

Construction : - Join PQ

Proof : - $\angle PAB + \angle PAQ = 180^\circ$ (Linear pair)

$$\angle PAQ + 125^\circ = 180^\circ$$

$$\angle PAQ = 180^\circ - 125^\circ$$

$$\angle PAQ = 55^\circ$$

Since the length of tangent from an external point to a circle are equal.

$$PA = QA$$

\therefore From $\triangle PAQ$

$$\angle APQ = \angle AQP$$

In $\triangle APQ$

$$\angle APQ + \angle AQP + \angle PAQ = 180^\circ \text{ (angle sum property)}$$

$$\angle APQ + \angle AQP + 55^\circ = 180^\circ$$

$$2\angle APQ = 180^\circ - 55^\circ \text{ } (\because \angle APQ = \angle AQP)$$

$$\angle APQ = \frac{125^\circ}{2}$$

$$\therefore \angle APQ = \angle AQP = \frac{125^\circ}{2}$$

OQ and OP are radii

QA and PA are tangents

$$\therefore \angle OQA = 90^\circ$$

$$\& \angle OPA = 90^\circ$$

$$\angle OPQ + \angle QPA = \angle OPA = 90^\circ \text{ (Linear Pair)}$$

$$\angle OPQ + \frac{125^\circ}{2} = 90^\circ$$

$$\begin{aligned} \angle OPQ &= 90^\circ - \frac{125^\circ}{2} \\ &= \frac{180^\circ - 125^\circ}{2} \end{aligned}$$

$$\angle OPQ = \frac{55^\circ}{2}$$

$$\text{Similarly } \angle OQP + \angle PQA = \angle OQA$$

$$\angle OQP + \frac{125^\circ}{2} = 90^\circ$$

$$\angle OQP = 90^\circ - \frac{125^\circ}{2}$$

$$\angle OQP = \frac{55^\circ}{2}$$

In $\triangle POQ$

$$\angle OQP + \angle OPQ + \angle POQ = 180^\circ \text{ (angle sum property)}$$

$$\frac{55^\circ}{2} + \frac{55^\circ}{2} + \angle POQ = 180^\circ$$

$$\angle POQ + \frac{110}{2} = 180^\circ$$

$$\angle POQ = 180^\circ - \frac{110}{2}$$

$$\angle POQ = \frac{360^\circ - 110^\circ}{2}$$

$$\angle POQ = \frac{250^\circ}{2}$$

$$\angle POQ = 125^\circ$$

$$\therefore \angle POQ = 125^\circ$$

7. Two tangents PA and PB are drawn to the circle with center O, such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$.

Ans: Given : - $\angle APB = 120^\circ$

Construction : - Join OP

To prove : - $OP = 2AP$

Proof :- $\angle APB = 120^\circ$

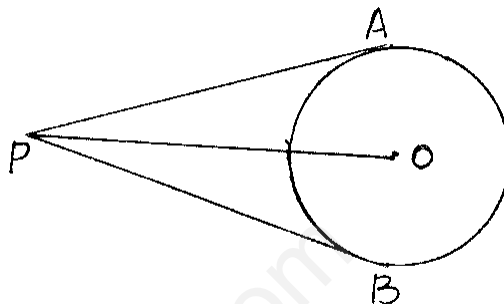
$$\therefore \angle APO = \angle OPB = 60^\circ$$

$$\cos 60^\circ = \frac{AP}{OP}$$

$$\frac{1}{2} = \frac{AP}{OP}$$

$$\therefore OP = 2AP$$

Hence proved



8. From a point P, two tangents PA are drawn to a circle with center O. If $OP = \text{diameter}$ of the circle show that triangle APB is equilateral.

Ans: $PA = PB$ (length of tangents from an external point)

From $\triangle OAP$,

$$\sin \angle APO = \frac{OA}{OP} = \frac{1}{2}$$

Since $OP = 2OA$ (Since $OP = \text{Diameter}$)

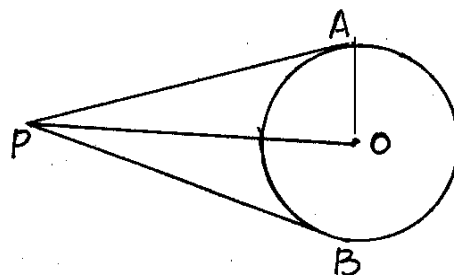
$$\therefore \angle APO = 30^\circ$$

since $\triangle APO \cong \triangle BPO$

$$\angle APO = \angle BPO = 30^\circ$$

$$\therefore \angle APB = 60^\circ$$

$\triangle APB$ is equilateral



9. In the given fig OPQR is a rhombus, three of its vertices lie on a circle with centre O. If the area of the rhombus is $32\sqrt{3} \text{ cm}^2$. Find the radius of the circle.

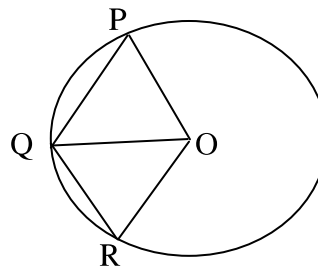
Ans: $QP = OR$

$OP = OQ$

$\therefore \triangle OPQ$ is an equilateral \triangle .

area of rhombus = 2 (ar of $\triangle OPQ$)

$$32\sqrt{3} = 2 \left(\frac{\sqrt{3}r^2}{4} \right)$$



$$32\sqrt{3} = \frac{\sqrt{3}r^2}{2}$$

$$r^2 = 32 \times 2 = 64$$

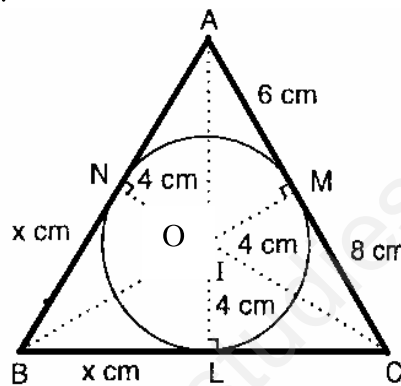
$$\Rightarrow r = 8\text{cm}$$

$$\therefore \text{Radius} = 8\text{cm}$$

10. If PA and PB are tangents to a circle from an outside point P, such that PA=10cm and $\angle APB=60^\circ$. Find the length of chord AB.

Self Practice

11. The radius of the in circle of a triangle is 4cm and the segments into which one side is divided by the point of contact are 6cm and 8cm. Determine the other two sides of the triangle.



(Ans: 15, 13)

Ans: $a = BC = x + 8$
 $b = AC = 6 + 8 = 14\text{cm}$
 $c = AB = x + 6$

$$\text{Semi-perimeter} = \frac{a+b+c}{2}$$

$$= \frac{BC + AC + AB}{2}$$

$$= \frac{x+8+14+x+6}{2}$$

$$= \frac{2x+28}{2}$$

$$= x+14$$

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} \text{ on substituting we get}$$

$$= \sqrt{(x+14)(6)(x)(8)}$$

$$= \sqrt{(x+14)(48x)} \dots\dots\dots(1)$$

Area of ΔABC = area ΔAOB + area ΔBOC + area ΔAOC

$$\text{area } \Delta AOC = \left(\frac{1}{2} b.h \right) = \frac{1}{2} \times 4 \times 14$$

$$= 28$$

On substituting we get

$$\therefore \text{area } \Delta ABC = \text{area } \Delta AOC + \text{area } \Delta BOC + \text{area } \Delta AOB$$

$$= 4x + 56 \dots\dots\dots(2)$$

From (1) and (2)

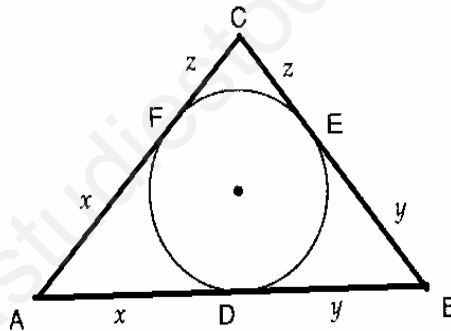
$$4x + 56 = \sqrt{(x+14)(48x)}$$

Simplify we get $x = 7$

$$\therefore AB = x + 6 = 7 + 6 = 13\text{cm}$$

$$\therefore BC = x + 8 = 7 + 8 = 15\text{cm}$$

12. A circle is inscribed in a triangle ABC having sides 8cm, 10cm and 12cm as shown in the figure. Find AD, BE and CF. (Ans :7cm ,5cm,3cm)



Self Practice

13. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Since $\Delta ADF \cong \Delta DFC$

$$\angle ADF = \angle CDF$$

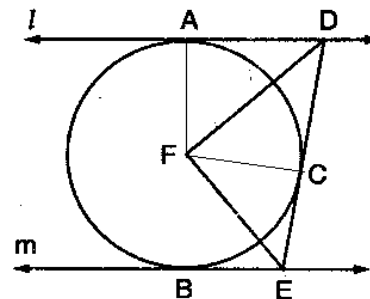
$$\therefore \angle ADC = 2 \angle CDF$$

Similarly we can prove $\angle CEB = 2 \angle CEF$

Since $l \parallel m$

$$\angle ADC + \angle CEB = 180^\circ$$

$$\Rightarrow 2\angle CDF + 2\angle CEF = 180^\circ$$



$$\Rightarrow \angle CDF + \angle CEF = 90^\circ$$

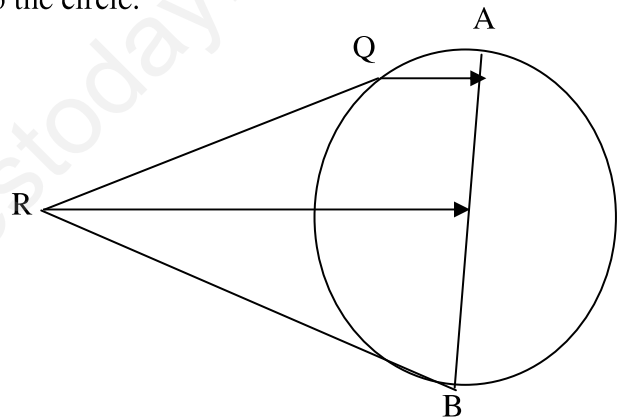
In $\triangle DFE$

$$\angle DFE = 90^\circ$$

14. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: Same as question No.5

15. QR is the tangent to the circle whose centre is P. If QA \parallel RP and AB is the diameter, prove that RB is a tangent to the circle.



Self Practice