Circles

1 Mark Question

Q1. Find the length of the tangent drawn from a point P.12 cm away from the centre of a circle of radius 5 cm.

Sol Length of tangent PA= $\sqrt{12^2-5^2}= \sqrt{119}$cm

Q2. Find the distance of A from the centre of circle.if the length of tangent from A to the circle of radius 9 cm is 12cm

Sol. In Δ ABO $\sqrt{12^2}+\sqrt{9^2}$=15 cm

Q3. Two tangents AB and CD common to two circles which touch each other at C. If point D lies on AB such that tangent CD equal to 4 cm then find the length of AB

Sol. AB= AD+BD

CD+CD= 2CD= 8cm

Q4 At how many points a secant of the circle intersect the circle

Sol. 2 Points

2 Mark Questions

Q1. Find the length of AP where O is the centre of circle BP is the tangent and OB is the radius

BP=8cm, OB=6cm

Sol. In Δ OPB

$BP^2=OB^2+OP^2$

$OP^2=8^2-6^2$

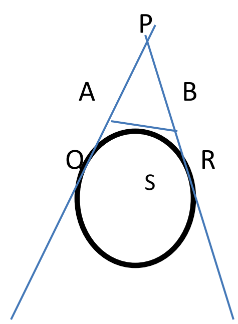
$OP=\sqrt{28}=2\sqrt{7}$

AP= OA+OP

$=OB+2\sqrt{7}$

$=6+2\sqrt{7}$cm

Q2. In the given figure PQ,PR and AB are tangents at points Q,R and S respectively of a circle. If PQ =8 cm .Find the Perimeter of triangle



Sol. AQ=AS

BR=BS

PQ=PR=8cm

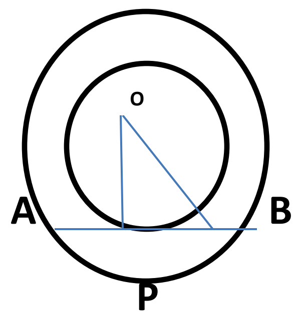
Perimeter of Δ APB =AP+AB+PB

= PQ-AQ+AS+BS+PR-BR

=PQ+PR

=8+8=16cm

Q3. In the figure AB is a chord of the outer circle of two concentric circles with centre O. If AB touches the inner circle at P OB=3cm, OP=5cm.Find AB



Sol. In Δ OPB,

$\llcorner$OPB=90°

$OB^2 = OP^2 + PB^2$

$(13)^2 = (5)^2 + (PB)^2$

PB= $\sqrt{169-25} = \sqrt{144} = 12$cm

AP=AB=12cm

AB=24cm

Q4. PQ is a tangent of length 6cm to the circle with centre O. If $\angle$OQP= 60° then find the length of OQ

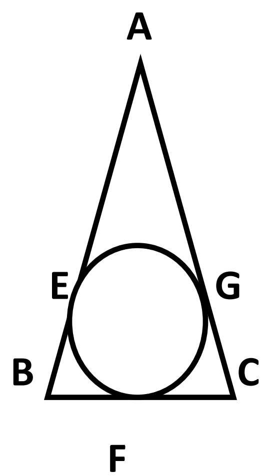
Sol. In rt. Δ OPQ cos Q =PQ/OQ   cos60°=6/OQ

1/2 =6/OQ

So, OQ=6x2=12cm

3 Mark Questions

Q1.If in the isosceles triangle ABC of the fig. given below AB=AC show that BF=FC.



Sol. In the fig. AB=AC (given)

Also, AE=AG

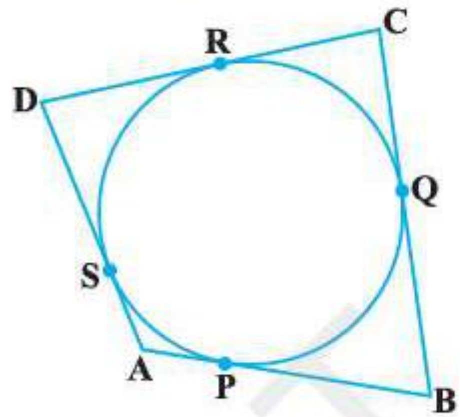
AB-AE= AC-AG

BE=CG 1

But BE=BF and CG=CF (Tangents)

From1... BF=CF

Q2.From the given fig. A Circle touches all four sides of Quadrilateral



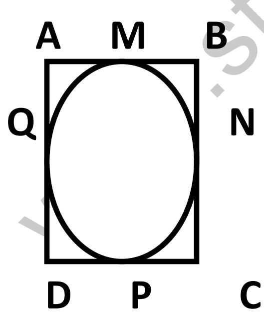
ABCD. Prove that AB+CD=BC+DA

Sol. From the fig. AS=AP, SD=DR , PB=BQ, CR=CQ(Tangents)

AS+SD+BQ+CQ=AP+PB+CR+DR

AD+BC=AB+CD

Q3.Prove that the ||gm circumscribing a circle isa rhombus



Sol. Given: ABCD ||gm touching the circle at M,N,P,Q

To prove: ABCD is a rhombus

Proof: AQ=AM

DQ=DP

BN=MB

NC=PC

Adding the above we get

AD+BC=AB+CD

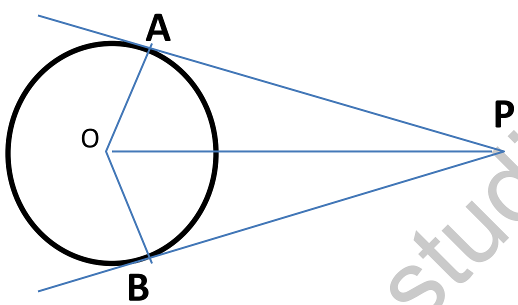
AD=BC and AB=CD

AD=AB=BC=CD

O O

It is a rhombus

Q4.Prove that tangents drawn from an external point are equally inclined to line joining the external point and the centre



Sol. In Δ PAO and ΔPBO

∠PAO=∠PBO

PO=OP

AO=OB

Δ PAO ≡ ΔPBO

Δ APO and ΔBPO

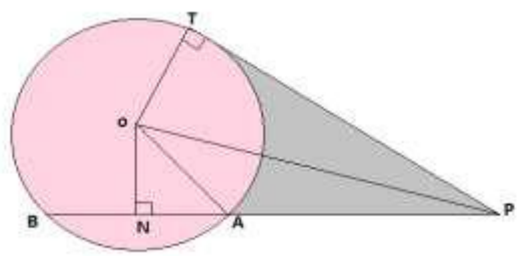
Hence proved.

4 Mark Questions

Q1.In figure 1, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on

the chord AB. Prove that

1. **$PA \times PB = PN^2 - AN^2$**
2. **$PN^2 - AN^2 = OP^2 - OT^2$**
3. **$PA \times PB = PT^2$**



Sol.. $PA \times PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$ ($\because$ BN=AN)

$= PN^2 - AN^2$

2. $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$ ($\because \angle ONP = 90°$)

$= OP^2 - (ON^2 + AN^2)$

$= OP^2 - OA^2$ (∵∠ONA= 90°)

$= OP^2 - OT^2$ (∵ OA=OT=radius)

3. From 1 & 2

$PA \times PB = OP^2 - OT^2 = PT^2$ (∵∠OTP= 90°)

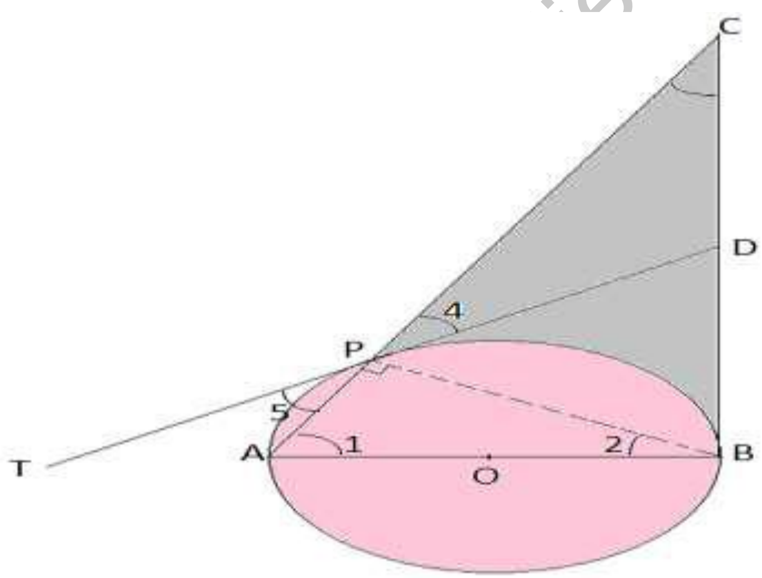
$\Rightarrow PA \times PB = PT^2$

3. From 1 & 2

$PA \times PB = OP^2 - OT^2 = PT^2$ (∵∠OTP= 90°)

$\Rightarrow PA \times PB = PT^2$

Q2.In a right triangle ABC (figure 2), in which ∠B= 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.

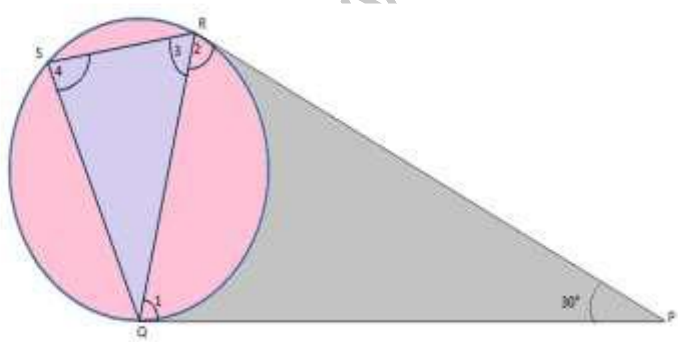


Sol.ABC is Right angled Δ with ∠ABC= 90°. Circle described on AB as diameter cuts AC at P and tangent to the circle at P intersect BC in D. We shall prove that D is the midpoint of BC.

Here ,∠1+ ∠2=90° (∵∠APB=90°,angle in a semicircle)
also ∠1+ ∠3=90° (∵∠ABC=90°)
⇒∠1+ ∠2 = ∠1+ ∠3
⇒∠2 = ∠3 ………………………….. (i)
also ∠2 = ∠5 (∵ angle in alternate segment)
and ∠5 = ∠4 (vertically opposite angles)
⇒∠2 = ∠4 ………………………(ii)

from (i) & (ii) ,
∠3 = ∠4
⇒ CD = PD
But PD = BD (Tangents from an external point are equal)
⇒ CD = BD
⇒ D is the midpoint of BC
⇒ The tangent at P bisects BC .

Q3.In given figure 3,tangents PQ and PR are drawn to a circle such that ∠RPQ=30°. A Chord RS is drawn parallel to the tangent PQ. Find ∠RQS.



Sol.Since PQ=PR

∴⇒∠1= ∠2

But ∠1 + ∠2 = 180° - 30° = 150°

⇒∠1= ½(150°) = 75°

⇒∠3 = ∠1 (Alternate angles as SR || PQ)

⇒∠3 = 75°…………………….(i)

Also ∠4= ∠1 (Angles in alternate segment)

⇒∠4 = 75°…………………….(ii)

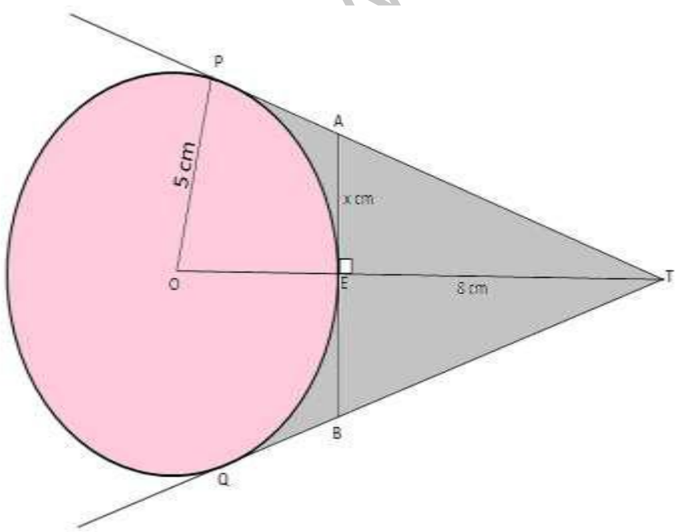
Since ∠RQS+ ∠3 + ∠4= 180°

Therefore, ∠RQS=180°- ∠3- ∠4

= 180°-75°-75° = 30°

Q4.In figure 4, O is the centre of a circle of radius 5cm. T is a point such that OT=13 cm and OT intersects the circle at E. If AB is the tangent to

the circle at E, find the length of AB. Here, A and B lie on the tangents to the circle drawn from T.



Sol.As OP $\perp$ PT $\therefore \angle OPT = 90°$

$\Rightarrow$ $PT^2 = OT^2 - OP^2 = (13^2 - 5^2)\ cm^2 = 144\ cm^2$

$\Rightarrow$ PT = 12 cm

Also, ET = OT – OE = 13 cm - 5 cm = 8cm

Let AE = x cm , then AP = AE = x cm

Hence , AT =PT - AP = (12-x) cm

Since , OE (or OT) $\perp$ AB, $\therefore \angle AET = 90°$

$\Rightarrow$ $AT^2 = AE^2 + ET^2$

$\Rightarrow$ $(12-x)^2 = x^2 + 8^2$

$\Rightarrow$ $144 + x^2 - 24x = x^2 + 64$

$\Rightarrow$ $-24x = 64 - 144$

$\Rightarrow$ $-24x = -80$

$\Rightarrow$ $x = -80/-24 = 10/3$

Hence AE= 10/3 cm

Consequently AB= 2AE= 2 × (10/3) cm = 20/3 cm ($\because$ AE=BE)