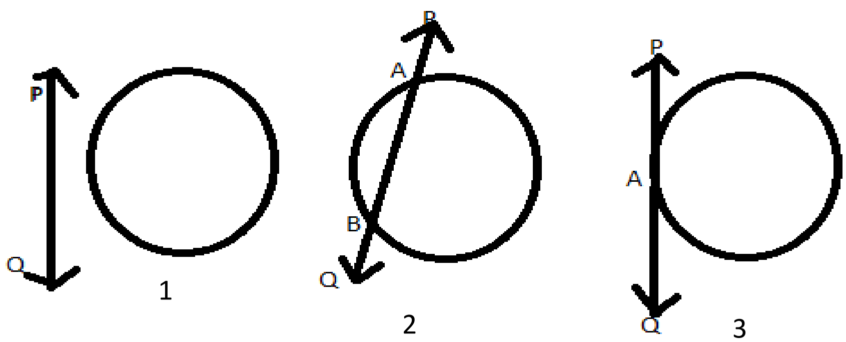
CIRCLES

Key Points

1. **Circle:** A circle is a collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre).

2. **Secant & Tangent to a Circle:** In fig. 1 the line PQ and the circle have no common point. Line PQ is called non-intersecting. In fig. 2 line PQ a secant to a circle. In fig. 3, there is only 1 point A, which is common to the line PQ and the circle. The line is called a tangent to the circle.



3. **Tangent to a Circle** :
   It is a line that intersects the circle at only one point. There is only one tangent at a point of the circle. The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

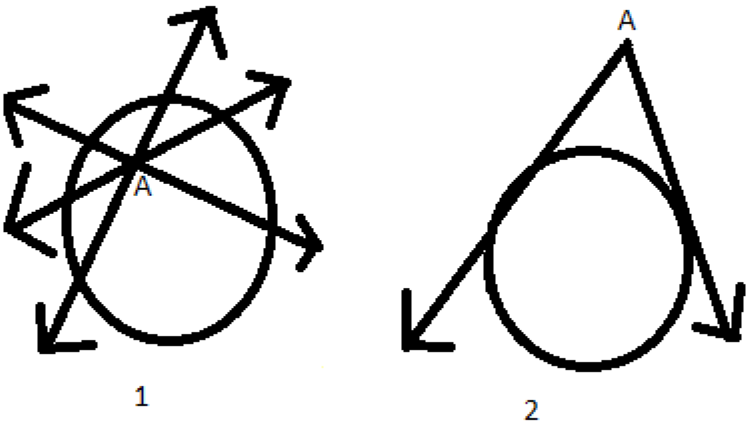
4. **Theorems :**
   1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
   2. The length of tangents drawn from an external point to a circle are equal.

5. Number of tangents from a point on a circle-

(i)There is no tangent to a circle passing through a point lying inside the circle.

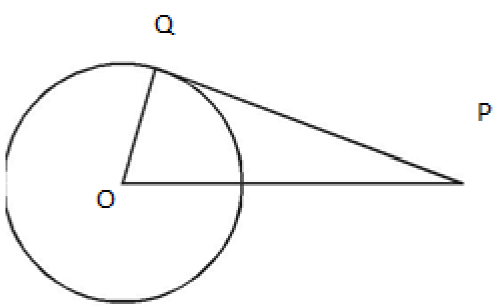
(ii)There is one and only one tangent to a circle passing through a point lying on the circle.

(iii)There are exactly two tangents to a circle through a point lying outside the circle.



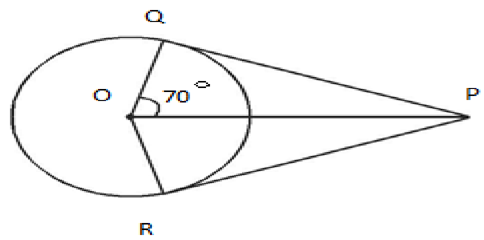
LEVEL I

1. In the given fig. O is the centre of the circle and PQ is tangent then $\angle POQ + \angle QPO$ is equal to

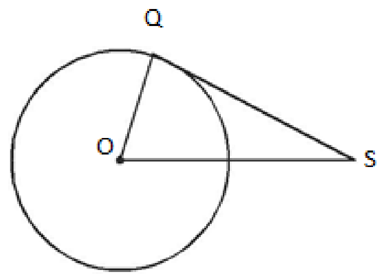


2. If PQ is a tangent to a circle of radius 5cm and PQ = 12 cm, Q is point of contact, then OP is

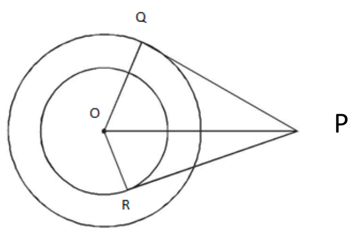
3. In the given fig. PQ and PR are tangents to the circle, $\angle QOP$ = 70°, then$\angle QPR$ is equal to



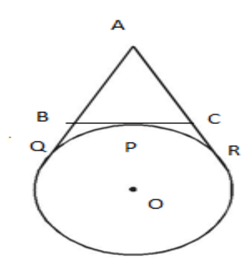
4. In the given fig. QS is a tangent to the circle, OS = 8 cm, OQ = 6 cm then the length of QS is



5. In the given fig PQ is tangent to outer circle and PR is tangent to inner circle. If PQ = 4 cm, OQ = 3 cm and OR = 2 cm then the length of PR is



6. In the given fig. P, Q and R are the points of contact. If AB = 4 cm, BP = 2 cm then the perimeter of ΔABC is

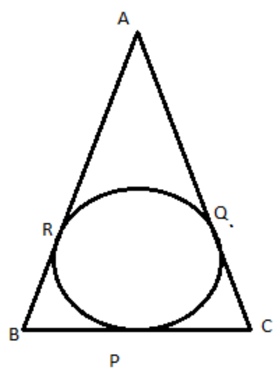


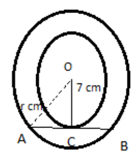
7. The distance between two tangent parallel to each other to a circle is 12 cm. The radius of circle is
8. The chord of a circle of radius 10cm subtends a right angle at its centre.Find the length of the chord.
9. How many tangents can a circle have?

10. How many tangents can be drawn from a given point to a circle?

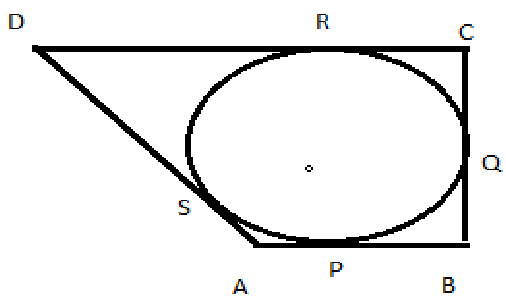
LEVEL - II

11. Two concentric circles of radii a & b (a>b) are given. Find the length of the chord of the larger circle which touches the smaller circle
12. From a point P outside the circle with centre O, tangents PA and PB are drawn to the circle. Prove that OP is the right bisector of the line segment AB.
13. A circles is inscribed in a triangle ABC, touching BC, CA and AB at P,Q and R respectively if AB = 10 cm AQ =7cm CQ =5 cm. Find BC





14. A Quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC



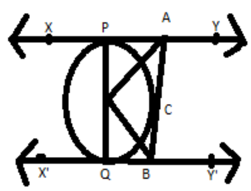
15. Two concentric circles are of radii 7 cm and r cm respectively, where r>7. A chord of the larger circle of length 46 cm, touches the smaller circle. Find the value of r.

16. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

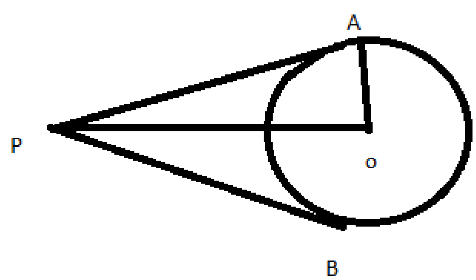
LEVEL - III

17. Prove that the length of tangents drawn from an external point to a circle are equal.

18. Prove that the tangents at the extremities of any chord of a circle, make equal angle with the chord.
19. PA and PB are tangents to the circle with the centre O from an external point P,touching the circle at A and B respectively. Show that the quadrilateral AOBP IS cyclic.
20. Prove that the parallelogram circumscribing a circle is a rhombus.
21. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that LAOB = 90^0.



Q.22 Two roads starting from P are touching a circular path at A and B. Sarita runs from P to A, 20km and A to O, 15km and Reeta runs from P to O directly. (Value based question)
(a) Find the distance covered by Reeta.
(b) Who will win the race?
(c) What value is depicted by Reeta?



SELF EVALUATION

1. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.
2. Prove that perpendicular at the point of contact to the tangent to a circle passes through the centre.

3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.
4. The length of a tangent from a point A at a distance 5cm from the centre of the circle is 4cm. Find the radius of the circle. Ans 12cm
5. Two concentric circles are of radii 6.5cm and 2.5cm. Find the length of the chord of larger circle which touches the smaller circle. Ans 3cm
6. From a point P, 10cm away from the centre of the circle, a tangent PT of length 8cm is drawn. Find the radius of the circle. Ans 6cm

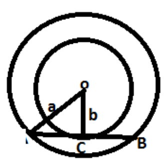
MARKING SCHEME

LEVEL-I

1. 90°
2. $\sqrt{119}$ cm
3. 40°
4. $\sqrt{28}$cm
5. $\sqrt{21}$ cm
6. 12 cm
7. 6cm
8. $10\sqrt{2}$cm
9. Infinite
10. Only 2 Tangents

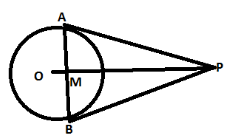
LEVEL-II

11. In Right ΔACO,



$OA^2 = OC^2 + AC^2$
AC = $\sqrt{a^2 - b^2}$
AB=2AC=$2\sqrt{a^2 - b^2}$ [C is midpoint of AB]

12. In ΔMAP and ΔMBP,
PA=PB [Tangents are equal)
MP=MP(Common)
<MPA ≅ <MPB (By SAS Congruence rule)



So, MA=MP [CPCT]
And <AMP= <BMP {CP CT}
BU <AMP+<BMP=180^0 [Linear Pair]
<AMP=<BMP=90^0

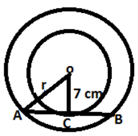
13.AR=AQ=7cm
BR= (AB-AR) = (10-7) cm = 3cm
BP = BR = 3 cm
CP = CQ=5 cm
BC=BP+CP= (3+5) cm = 8cm

14.AP=AS------------------ (I) [Tangents from A]
BP = BQ------------------ (II) [Tangents from B]
CR=CQ------------------ (III) [Tangents from C]
DR=DS------------------ (IV) [Tangents from D]
AB+CD= (AP+BP) + (CR+DR)
(AS+BQ)+ (CQ+DS) (USING I, II, III, IV)
= (AS+DS) + (BQ+CQ)
=AS+BC
Hence, AB+CD=AD+BC

15.ΔACO we have,

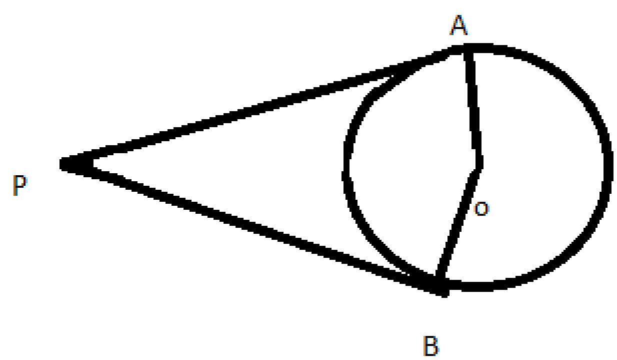
$OA^2=OC^2+AC^2$ [By Pythagoras Theorem]
OA=$\sqrt{(OC)^2+(AC)^2}$
r =$\sqrt{(OC)^2+(1/2AB)^2}$ [C is mid-point of AB]
r =$\sqrt{7^2+23^2}$
r =$\sqrt{578}$

r =$17\sqrt{2}$ CM

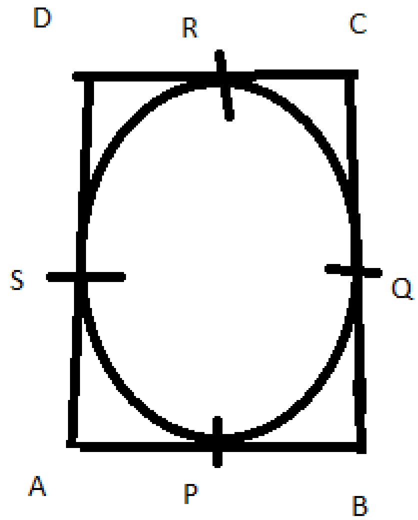


Level III

17. Correct construction
Figure
Proof
18. Correct construction
Figure
Proof
19.



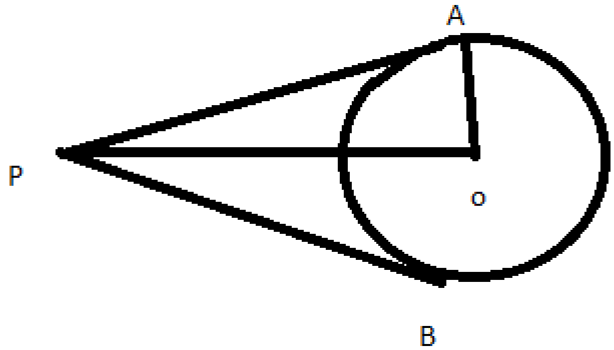
Quad. OAPB,
L AOB + L OAP + L APB + L OBP = 360^0
Or, L AOB +90^0 + L APB +90^0 = 360^0
Or, L AOB + L APB + 180^0 = 360^0
Or, L AOB + L APB = 180^0
Hence, quad. OAPB is cyclic.
20.



AP=AS ....(i) [Tangents from A]
BP = BQ ....(ii) [Tangents from B]
CR = CQ ....(iii) [Tangents from c]
DR = DS ....(iv) [Tangents from D]
Now, AB + CB = AP + BP + CR + DR
= AS + BQ + CQ + DS [From (i), (ii), (iii), (iv)]
= (AS + DS) + (BQ + CQ)
= AD + BC
Or, AB + CD = AD + BC
Or, 2AB = 2AD
Or, AB = AD
Hence, AB = BC = CD = AD
Hence, ABCD is a rhombus.
21.In quad. APQB
L APO + L BQO + L QBC + L PAC = 360^0
Or, 90^0 +90^0 + L QBC + L PAC = 360^0
Or, L QBC + LPAC = 180^0 ....(i)
We have, L CAO = ½ L PAC
And L CBO = ½ L QBC
Now,L CAO + L CBO = ½ (L PAC + L QBC)
= ½ X 180^0 (from eq. i)
=90^0.................................. (ii)
In triangle AOB,
L CAO + LAOB + L CBO = 180^0
Or, L AOB + 90^0 = 180^0 (from eq. ii)

Or, L AOB = 90^0

22.(i)



In triangle OAP,

$OP^2 = OA^2 + AP^2$ (By Pythagoras Theorem)

Or, $OP^2 = (15)^2 + (20)^2$

Or, $OP^2 = 625$

Or, OP = 25 km

(ii) Distance covered by Rita = 25 km

Distance covered by Sarita = 20 km +15 km

= 35 km

So, Rita will win the race.

(iii) Rita chooses shortest path to reach at O.

So, it shows her intelligence.