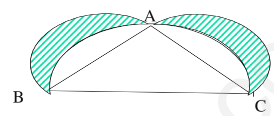
AREAS RELATED TO CIRCLES

The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful.

1. In the adjoining figure ΔABC right angled triangle right angled at A. Semi circles are drawn on the sides of the triangle ΔABC. Prove that area of the Shaded region is equal to area of ΔABC



Ans: Refer CBSE paper 2008

2. The sum of the diameters of two circles is 2.8 m and their difference of circumferences is 0.88m. Find the radii of the two circles (Ans: 77, 63)

Ans: $d_1 + d_2 = 2.8$ m= 280cm

$r_1+r_2 = 140$

$2\ \Pi\ (r_1 - r_2) = 0.88m = 88cm$

$r_1 - r_2 = \frac{88}{2\Pi} = \frac{88x7}{44} = 2 \text{ x } 7 = 14$

$r_1+r_2 = 140$

$r_1-r_2 = 14$

$2r_1 = 154$

$r_1=77$

$r_2 = 140 - 77 = 63$

$r_1 = 77$ cm, $r_2= 63$cm

3 Find the circumference of a circle whose area is 16 times the area of the circle with diameter 7cm (Ans: 88cm)

Ans: $\Pi R^2 = 16\ \Pi r^2$

$R^2 = 16\, r^2$

$R^2 = 16 \text{ x } \frac{7}{2} \text{ x } \frac{7}{2}$

$= 49 \text{ x } 4 \quad \Rightarrow R = 7 \text{ x } 2 = 14cm$

Circumference $= 2 \text{ x } \frac{22}{7} \text{ x } 14 = 2 \text{ x } 22 \text{ x } 2 = 88$ cm

4. Find the area enclosed between two concentric circles of radii 3.5cm, 7cm. A third concentric circle is drawn outside the 7cm circle so that the area enclosed between it and the 7cm circle is same as that between two inner circles. Find the radius of the third circle (Ans: 115.5 cm^2 $r = \sqrt{343}/2$)

Ans: Area between first two circles = $\Pi \times 7^2 - \Pi \times 3.5^2$

$= 49\Pi - 12.25\Pi$ -------------(1)

Area between next two circles = $\Pi r^2 - \Pi \times 7^2$

$= \Pi r^2 - 49\Pi$ -----------------(2)

(1) & (2) are equal

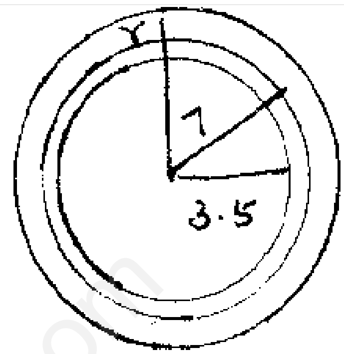
$49\Pi - 12.25\Pi = \Pi r^2 - 49\Pi$

$\Pi r^2 = 49\Pi + 49\Pi - 12.25\Pi$

$\therefore r^2 = 98 - 12.25 = 85.75$

$r^2 = \frac{8575}{100} = \frac{343}{4}$

$r = \frac{\sqrt{343}}{2}$ cm.



5. Two circles touch externally. The sum of their areas is 58π cm^2 and the distance between their centres is 10 cm. Find the radii of the two circles. (Ans:7cm, 3cm)

Ans: Sum of areas = $\Pi r^2 + \Pi(10 - r)^2 = 58\Pi$

$\Pi r^2 + \Pi(100 - 20r + r^2) = 58\Pi$

$r^2 + 100 - 20r + r^2 = 58$

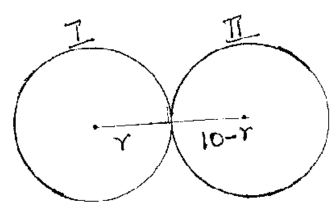
$2r^2 - 20r + 100 - 58 = 0$

$2r^2 - 20r + 42 = 0$

$r^2 - 10r + 21 = 0$

$(r-7), (r-3) = 0$

r=7cm,3cm



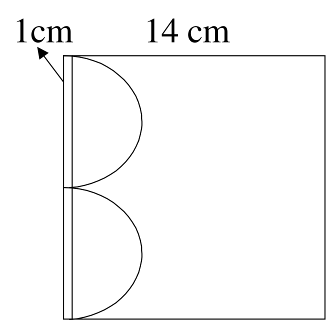
6. From a sheet of cardboard in the shape of a square of side 14 cm, a piece in the shape of letter B is cut off. The curved side of the letter consists of two equal semicircles & the breadth of the rectangular piece is 1 cm. Find the area of the remaining part of cardboard. (Ans: 143.5 cm^2)

Ans: Area of remaining portion = Area of square – Area of 2 semi circles – Area of rectangle

$= 14 \times 14 - \Pi \times 3.5^2 - 14 \times 1$

$= 196 - \frac{22}{7} \times 3.5 \times 3.5 - 14$

$= 196 - 38.5 - 14 = 143.5$ cm^2



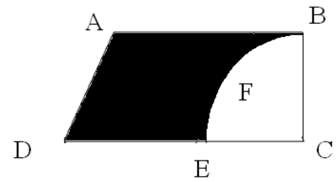
7. A piece of cardboard in the shape of a trapezium ABCD & AB ∥ DE, $\angle BCD = 90^0$, quarter circle BFEC is removed. Given AB = BC = 3.5 cm, DE = 2 cm. Calculate the area of remaining piece of cardboard. (Ans:6.125 cm^2)

Ans: Area of remaining portion = Area of trap – Area of quadrant

$$= \frac{1}{2} \text{ x } 3.5\,(5.5 + 3.5) - \frac{1}{4} \text{ x } \frac{22}{7} \text{ x}3.5\text{x}3.5$$

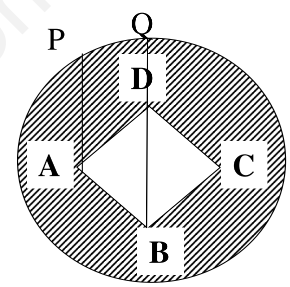
$$= 15.75 - \frac{19.25}{2} = 15.75 - 9.625$$

$$= 6.125 \text{ cm}^2$$



8. In the figure, ABCD is a square inside a circle with centre O. The Centre of the square coincides with O & the diagonal AC is horizontal of AP, DQ are vertical & AP = 45 cm, DQ = 25 cm. Find a) the radius of the circle b) si

c) area of shaded region (use π =3.14 , $\sqrt{2}$ = 1.41)



Ans: a) 53cm
b) 39.48cm
c) 7252.26 cm^2

Self Practice

9. The area enclosed between two concentric circles is 770cm^2. If the radius of the outer circle is 21cm, find the radius of the inner circle. (Ans :14cm)

Ans:

$$\Pi R^2 - \Pi r^2 = 770$$

$$\Pi (21^2 - r^2) = 770$$

$$21^2 - r^2 = \frac{770}{22} \text{ x } 7 = \frac{70}{2} \text{ x } 7$$

$$r^2 = 441 - \frac{490}{2} = 441 - 245 = 196$$

$$r = \pm 14$$

$$r = 14\text{cm}$$

10. A circular disc of 6 cm radius is divided into three sectors with central angles 120^0, 150^0,90^0. What part of the circle is the sector with central angles 120^0. Also give the ratio of the areas of three sectors. (Ans: $\frac{1}{3}$ (Area of the circle) 4 : 5 : 3)

Ans: Ratio of areas $= \frac{120}{360}\ \Pi \text{ x } 6^2 : \frac{150}{360}\ \Pi \text{ x } 6^2 : \frac{90}{360}\ \Pi \text{ x } 6^2$

$= 12\ \Pi : 15\Pi : 9\Pi$
$= 4 : 5 : 3$

Area of sector of central angle $120^o = \frac{120^o}{360^o} x\Pi\ r^2$

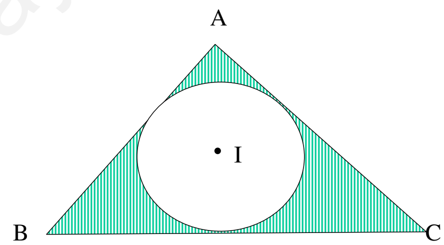
(i.e.) $\frac{1}{3}$ of area of the circle.

11. If the minute hand of a big clock is 1.05 m long, find the rate at which its tip is moving in cm per minute. (Ans:11cm/min)

Ans: Self Practice

12. ABC is a right angled triangle in which $\angle A = 90^0$. Find the area of the shaded region if AB = 6 cm, BC=10cm & I is the centre of the Incircle of ΔABC. (Ans: $\frac{80}{7}$ sq.cm)

Ans: $\angle A = 90^0$



BC = 10cm; AB = 6cm;
$\therefore$ AC = 6cm

Area of the $\Delta = \frac{1}{2}$ x 6 x 8= 24 cm^2

Let the Radius of the Incircle be r

$\therefore \frac{1}{2}$ x 10 x r + $\frac{1}{2}$ x 8 x r + $\frac{1}{2}$ x 6 x r = 24

$\frac{1}{2}$ r [10 + 8 + 6] = 24

r= 2 cm

$\therefore$ Area of circle = $\Pi r^2 = \frac{22}{7}$ x 2 x 2 = $\frac{88}{7}$ cm^2

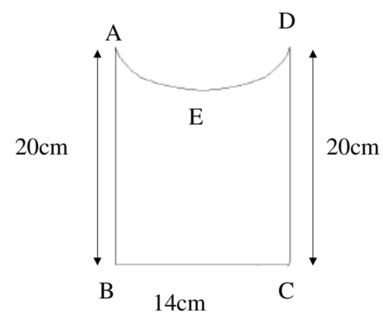
Area of shaded region = 24 - $\frac{88}{7} = \frac{168-88}{7} = \frac{80}{7}$ cm^2

13. Find the perimeter of the figure, where AED is a semi-circle and ABCD is a rectangle. (Ans : 76cm)

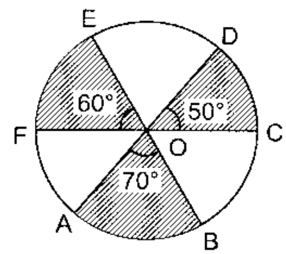
Ans: Perimeter of the fig = 20 + 14 + 20 + length of the arc (AED)

Length of Arc = (Π x r) = $\frac{22}{7}$ x7 = 22cm

$\therefore$Perimeter of the figure = 76 cm



14. Find the area of shaded region of circle of radius =7cm, if $\angle AOB=70^o$, $\angle COD=50^o$ and $\angle EOF=60^o$.



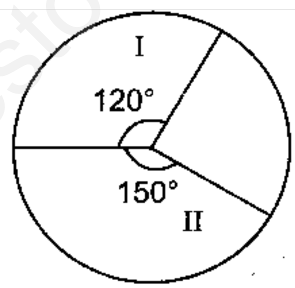
(Ans:$77 cm^2$)

Ans: Ar(Sector AOB + Sector COD + Sector OEF)

$$= \frac{70}{360} \Pi \times 7^2 + \frac{50}{360} \Pi \times 7^2 + \frac{60}{360} \Pi \times 7^2$$

$$49 \Pi \left(\frac{7}{36} + \frac{5}{36} + \frac{6}{36}\right) = 49 \Pi \times \frac{18}{36} = \frac{49}{2} \times \frac{22}{7} = 77 \text{ cm}^2$$

15. What is the ratio of the areas of sectors I and II ? (Ans:4:5)

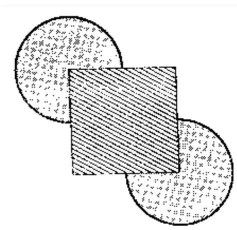


Ans: Ratio will be

$$\frac{120}{360} \Pi r^2 : \frac{150}{360} \Pi r^2$$

$$\frac{4}{12} : \frac{5}{12} = 4:5$$

16. Find the area of shaded region, if the side of square is 28cm and radius of the sector is ½ the length of side of square. (Ans:1708cm)



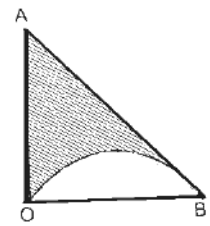
Ans: Area of shaded region is

$$2 \left(\frac{270}{360}\right) \Pi \times 14 \times 14 + 28 \times 28$$

$$2 \times \frac{3}{4} \times \frac{22}{7} \times 14 \times 14 + 784$$

$$924 + 784 = 1708 \text{ cm}^2$$

17. If OA = OB = 14cm, $\angle AOB = 90^o$, find the area of shaded region. (Ans:21cm^2)



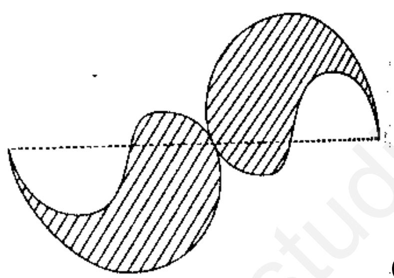
Ans: Area of the shaded region

= Area of Δ AOB – Area of Semi Circle

$= \frac{1}{2} \times 14 \times 14 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$98 - 77 = 21$ cm^2

18. The given figure consists of four small semicircles and two big semicircles. If the smaller semicircles are equal in radii and the bigger semicircles are also equal in radii, find the perimeter and the area of the shaded portion of the figure. Given that radius of each bigger semicircle is 42cm.



(Ans:528cm, 5544 sq cm)

Ans: Perimeter of the shaded region

= 2 [Perimeter (Bigger semi circle) + Perimeter (smaller semi circle) + Perimeter (small semi circle)]

$= 2 (42 \Pi + 21 \Pi + 21 \Pi)$

$= 84 \Pi$

$= 2 \times 84 \times \frac{22}{7} = 24 \times 22 = 528$ cm

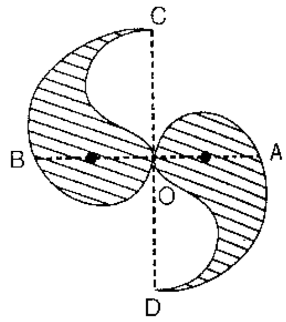
Area of shaded region

= [Area(big semi circle)]

$= 2 \times \Pi \times 42 \times 42 \times \frac{1}{2} = \frac{22}{7} \times 42 \times 42 = 5544$ cm^2

19. The boundary of the shaded portion in the adjoining figure consists of our half-circles and two quarter-circles. Find the length of the boundary and the area of the shaded portion, if OA=OB=OC=OD=14cm. (Ans:132 cm, 308 sq cm)

Ans: Proceed as in sum no 18.



20. The adjoining figure shows the cross-section of a railway tunnel.

The radius of the tunnel is 3.5m (i.e., OA=3.5m) and $\angle AOB = 90^o$.

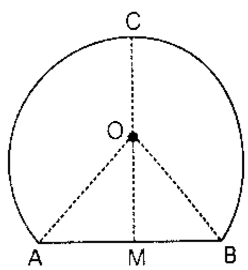
Calculate :

i. the height of the tunnel.

ii. the perimeter of its cross section, including base.

iii. the area of the cross-section

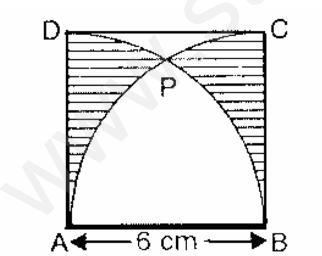
iv. the internal surface area of the tunnel, excluding base, if its length is 50m.



(Ans: (i) 5.97m (ii) 21.44m (iii) 28.875 sq m (iv) 825 sq m)

Ans: Self Practice

21. In the adjoining figure, ABCD is a square of side 6cm. Find the area of the shaded region.



(Ans: 34.428 sq cm)

Ans: From P draw $PQ \perp AB$

AQ = QB = 3cm

Join PB. Since arc APC is described by a circle with center B,

so BA = BP=BC =6cm.

In Δ PQB $\cos\theta = \frac{QB}{PB} = \frac{1}{2}$

$\therefore \theta = 60^o$

Area of sector BPA = $\frac{60}{360}\Pi(6^2) = 18.84$cm

Area of Δ BPQ = $\frac{1}{2}$(QB) (PQ) = $\frac{1}{2}$(3)(6 Sin 60) = 7.794Sq.cm

→ Area of portion APQ = Area of sector BPA – Area of Δ BPQ

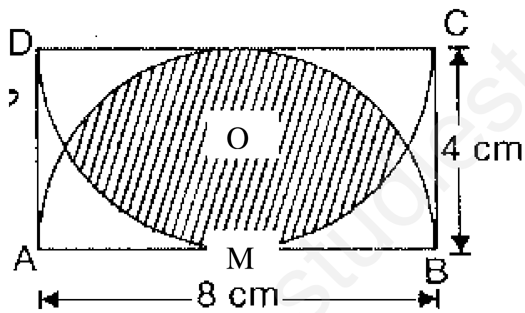
= 18.84 – 7.794 = 11.046 Sq.cm

Area of shaded portion = 2 x Area of Quadrant ABC – 2 Area APQ

$= [2 \times \frac{\Pi}{4}(6)^2 - 2 \times 11.046]$

=34.428 Sq.cm

22. In the adjoining figure, ABCD is a rectangle with sides 4cm and 8cm. Taking 8cm as the diameter, two semicircles are drawn. Find the area overlapped by the two semicircles.



(Ans:19.66 sq cm)

Ans: In Δ OMB

$\text{Cos}\angle \text{BOM} = \frac{OM}{OB} = \frac{2}{4} = \frac{1}{2}$

$\therefore \angle \text{BOM} = 60^0$

$\angle \text{AOB} = 120^0$

Area. Overlapped by semi circles

$= 2\left(\frac{120}{360} \times \Pi\ (4^2) - \frac{1}{2}\ \text{AB} \times \text{OM}\right)$

$= 2\left(\frac{\Pi}{3} \times 16 - \frac{1}{2}\ (2 \times \text{AM Sin } 60^0) \times 2\right)$

$= 2\left(\frac{22}{7} \times \frac{1}{3} \times 16 - 2 \times 4 \times \frac{\sqrt{3}}{2}\right)$